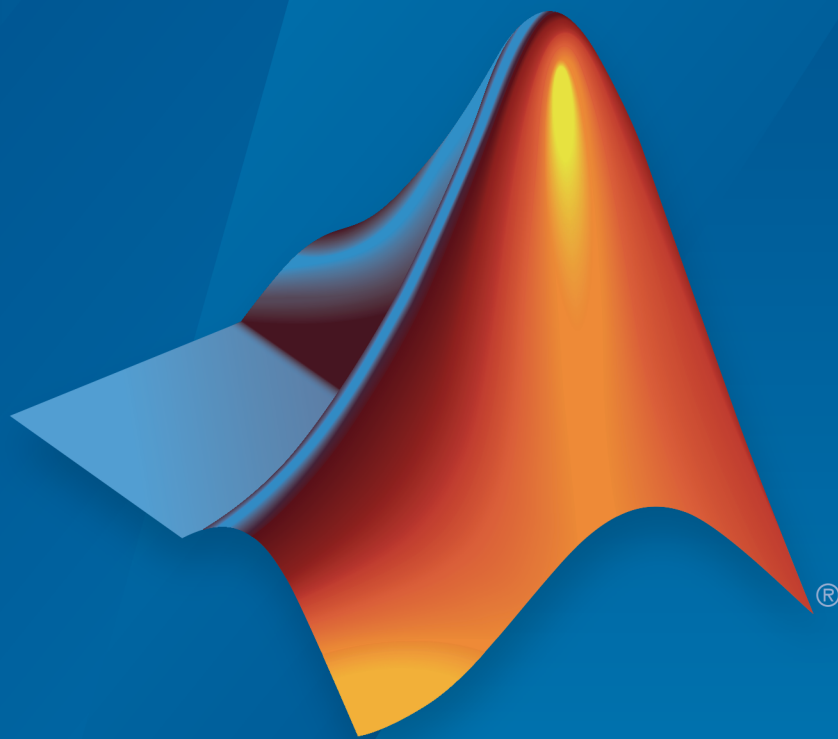# Antenna Toolbox™
User's Guide

MATLAB®

MathWorks®

How to Contact MathWorks

Latest news: www.mathworks.com

Sales and services: www.mathworks.com/sales_and_services

User community: www.mathworks.com/matlabcentral

Technical support: www.mathworks.com/support/contact_us

Phone: 508-647-7000

The MathWorks, Inc.
3 Apple Hill Drive
Natick, MA 01760-2098

Trademarks

Patents

Revision History

Contents

Array Concepts

2

Computational Techniques

3

Antenna Concepts

Port Analysis

In this section...

The *port* of an antenna is the physical location where the RF source is connected. From a network theory perspective, the antenna has a single port. In Antenna Toolbox™, a red dot on the antenna figure represents the feed point. A half-wavelength dipole is shown with its feed point:

dipole antenna element
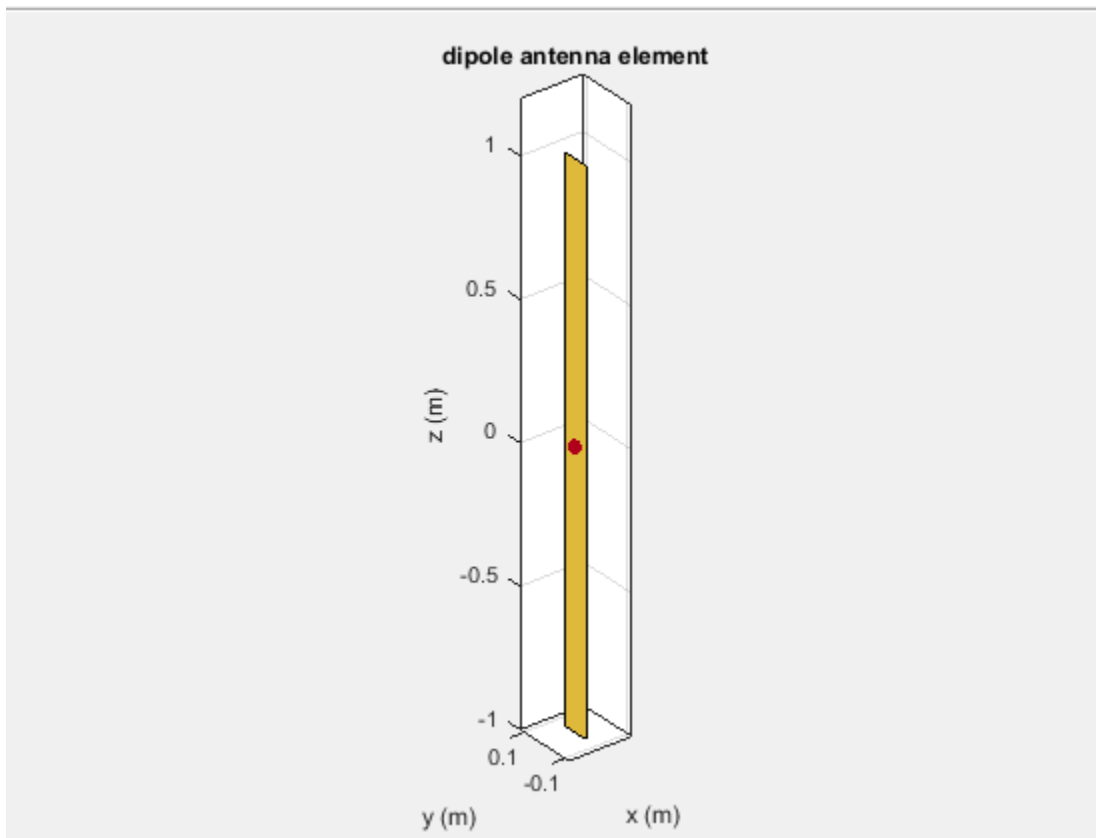
All antennas are excited by a voltage of 1V at the port. The various terminal port parameters are as follows:

Input Impedance

Input impedance is the ratio of voltage to current at the port. Antenna impedance is calculated as the ratio of the phasor voltage, which is 1V at a phase angle of 0 deg, to the phasor current at the port. The impedance equation is:

$$Z = V / I = R + jX$$

where:

- **V** is the antenna excitation voltage
- **I** is the current
- **R** is the antenna resistance in ohms
- **X** is the antenna reactance in ohms

Antenna input impedance is a frequency-dependent quantity. The plot shows the input impedance of a dipole antenna over the frequency band 20–120 MHz. The resistance and reactance traces vary with frequency. The variation can be qualitatively described in terms of resonances.

```
d = dipole;
impedance(d,20e6:1e6:120e6)
```
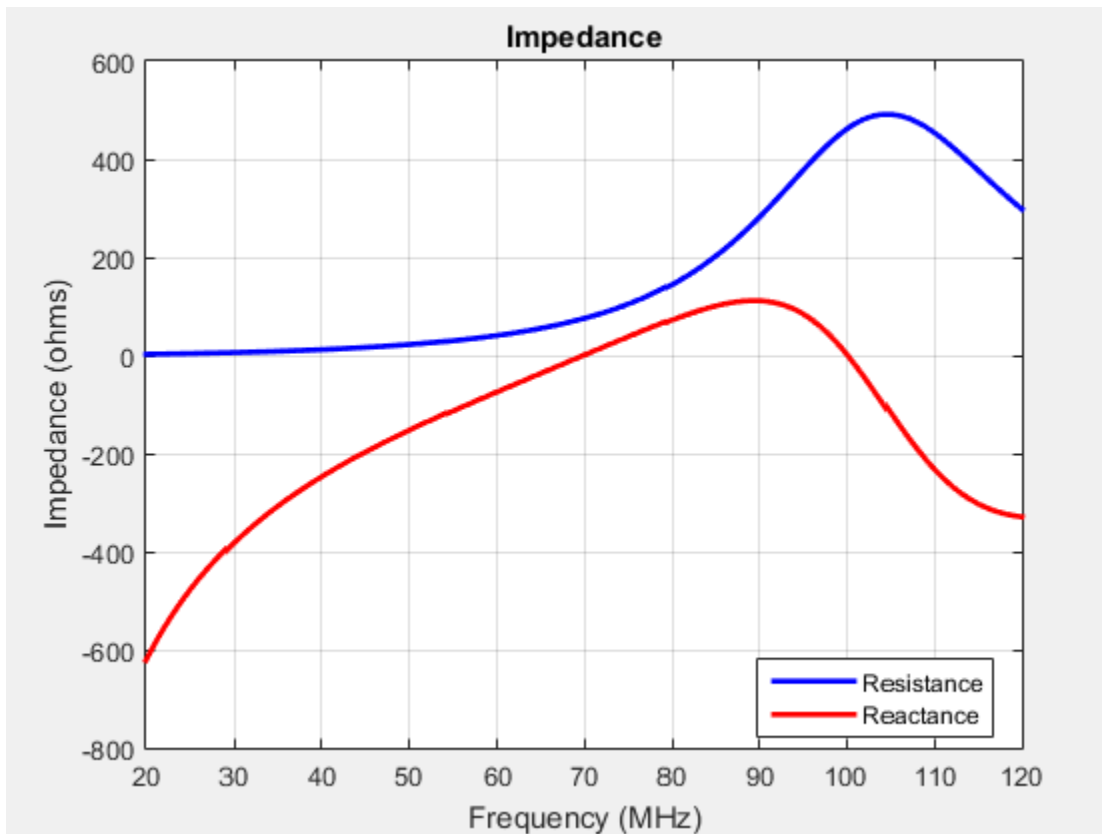
Use `impedance` to calculate the input impedance of any antennas in Antenna Toolbox.

Resonance

The *resonant frequency* of the antenna is the frequency at which the reactance of the antenna is equal to zero.

The plot shows two resonance points of a dipole antenna.
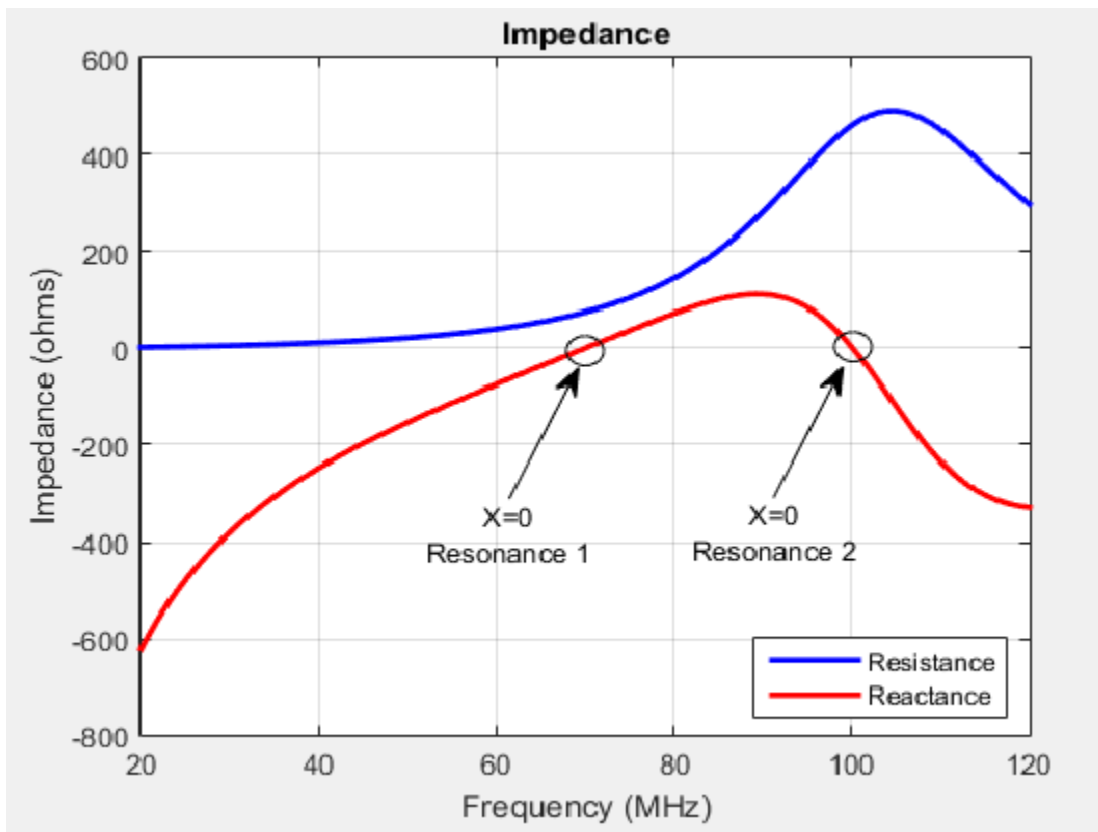
In the plot, the reactance values are negative, or capacitive, before the resonance. These values are positive or inductive after the resonance. This type of resonance is called series resonance. You can model this type of resonance using a series RLC circuit. If the impedance curve goes from positive reactance to negative reactance, it is called parallel resonance. You can model this type of resonance using a parallel RLC circuit.

Reflection Coefficient

The *reflection coefficient*, or S_1_1, of the antenna describes a relative fraction of the incident RF power that is reflected back due to the impedance mismatch. Impedance mismatch is the difference between the input impedance of the antenna and the characteristic impedance of the transmission line (or the generator impedance when the transmission line is not present). The characteristic impedance is the reference impedance.

```
S = sparameters(d,20e6:1e6:120e6,72)
rfplot(S)
```
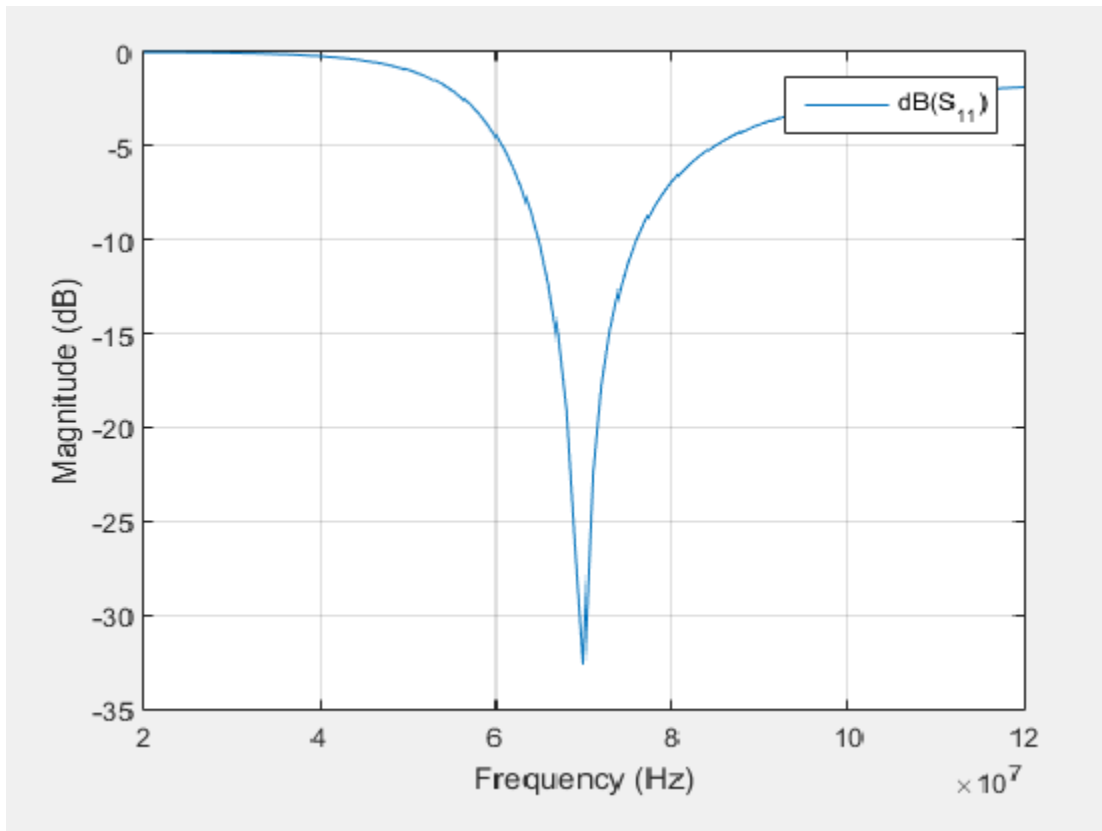
The reflection coefficient also gives the operating bandwidth of the antenna. Antenna bandwidth is usually the frequency band over which the magnitude of the reflection coefficient is below –10 dB.

Use `sparameters` to calculate the value of S_{11} for any antenna in the Antenna Toolbox.

Return Loss

The *return loss* of an antenna is a measure of the effectiveness of power delivery from a transmission line or coaxial cable to a load such as an antenna. The return loss can also be defined as the difference in dB between the power sent toward the antenna and the power reflected back from it. The higher the power ratio, the better matching between load and line. Return loss equation is:

$$RL = -20\log_{10}\left|S_{11}\right|$$

where:

- `RL` is the return loss
- S_{11} is the reflection coefficient, or power reflected from the antenna.

For passive devices, the return loss is a positive nondissipative term representing the reduction in amplitude of the reflected wave in comparison to the incident wave. In active devices, a negative return loss is possible.

```
d = dipole;
returnLoss(d,20e6:1e6:120e6,72)
```
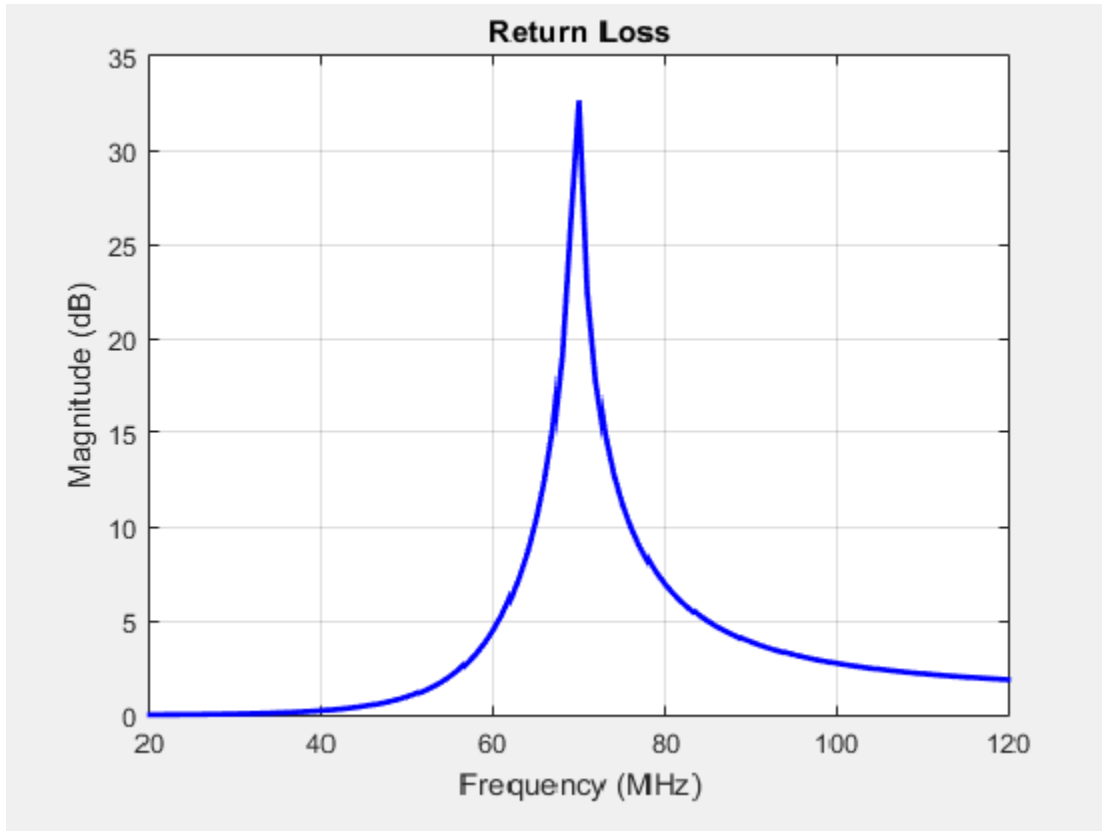
Return loss plots also give the operating bandwidth of the antenna. Antenna bandwidth is the frequency band over which the magnitude of return loss is greater than 10 dB. Use the returnLoss function to calculate the return loss of any antenna in the Antenna Toolbox library.

Voltage Standing Wave Ratio

The *voltage standing wave ratio* (VSWR) of an antenna is another measure of impedance matching between transmission line and antenna. The standing wave is generated because of the impedance mismatch at the port. VSWR equation is:

$$VSWR = \frac{1+|S_{11}|}{1-|S_{11}|}$$

where:

- S_{11} is the reflection coefficient.

```
d = dipole;
vswr(d,20e6:1e6:120e6,72)
axis([20 120 1 20])
```
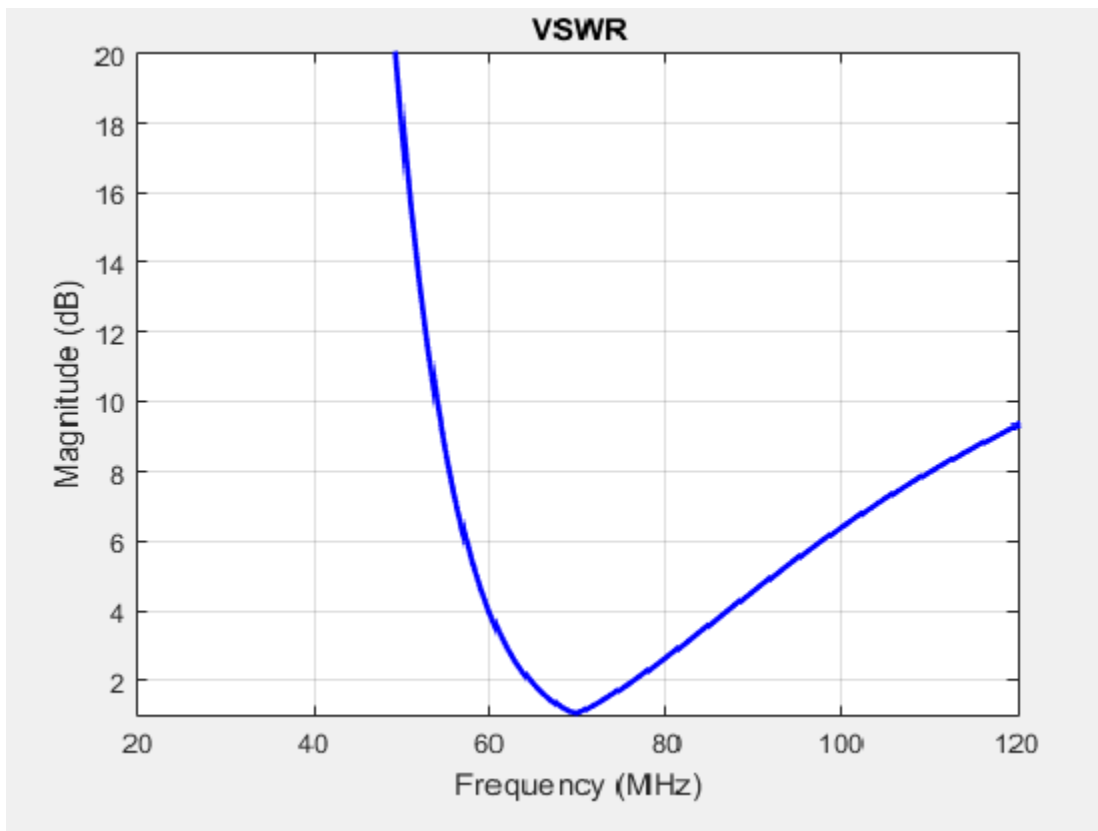

VSWR is scalar and contains no phase information. The value of VSWR lies between 1 and infinity. Antenna bandwidth is usually the frequency band over which the VSWR is less than approximately 2.

Use `vswr` to calculate the voltage standing wave ratio for any antenna in Antenna Toolbox.

Bandwidth

Bandwidth describes the range of frequencies over which the antenna can properly radiate or receive energy. It is a fundamental antenna parameter. Often, the desired bandwidth is one of the parameters used to determine which antenna to use. Antenna bandwidth is usually the frequency band over which the magnitude of the reflection coefficient is below -10 dB, or the magnitude of the return loss is greater than 10 dB, or the VSWR is less than approximately 2. All these criteria are equivalent. You can control the bandwidth using proper antenna design.

References

[1] Balanis, C.A. *Antenna Theory: Analysis and Design.*3rd Ed. New York: Wiley, 2005.

[2] Stutzman, Warren L., and Thiele, Gary A. *Antenna Theory and Design.* 3rd Ed. New York: Wiley, 2013.

[3] Bird, T.S. "Definition and Misuse of Return Loss." *IEEE Antennas and Propagation Magazine.* Vol. 51, Issue 2, April 2009, pp. 166–167.

Surface Analysis

Charge Distribution

The flow of charges on the antenna surface determines the surface currents of the antenna. For antennas to radiate, there must be acceleration or deceleration of charges. The deceleration of charges is caused due to buildup of charges at the end of the wire, which leads to impedance discontinuities. This mechanism creates electromagnetic radiation. The accumulation of charges varies according to time and structure of the antenna.

The accumulation of charges is exploited in many ways. If you calculate the impedance of this monopole antenna using the impedance function, you get the following plot:

```
m = monopole
impedance(m,20e6:1e6:120e6)
```
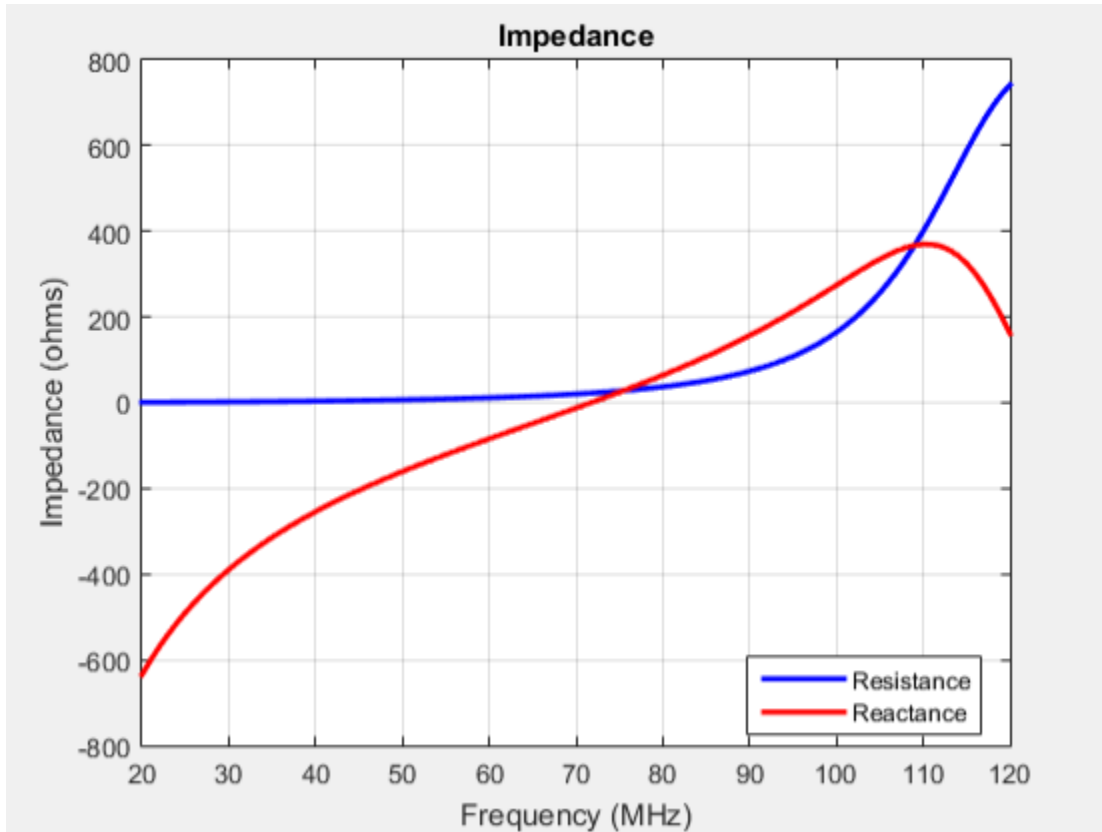
You can observe the first resonance is at approximately 71 MHz. To lower the resonance frequency, recalculate the height of the monopole to quarter wavelength. The frequency of operation is also lower. You must also have to increase the size of the corresponding ground plane. This increase in size means that to achieve similar performance at a lower frequency, you need a larger antenna. This approach is not possible due to physical space constraints.

Alternatively, you can exploit the fact that antennas have charge accumulation. If you provide appropriate structural modification to the antenna, charges accumulate. For a monopole antenna, you can enable charge accumulation by adding a top-hat to the monopole. Now, if you calculate the impedance of the antenna using the top-hat, the plot is:

```
mt = monopoleTopHat
```

```
impedance(mt,20e6:1e6:120e6)
```
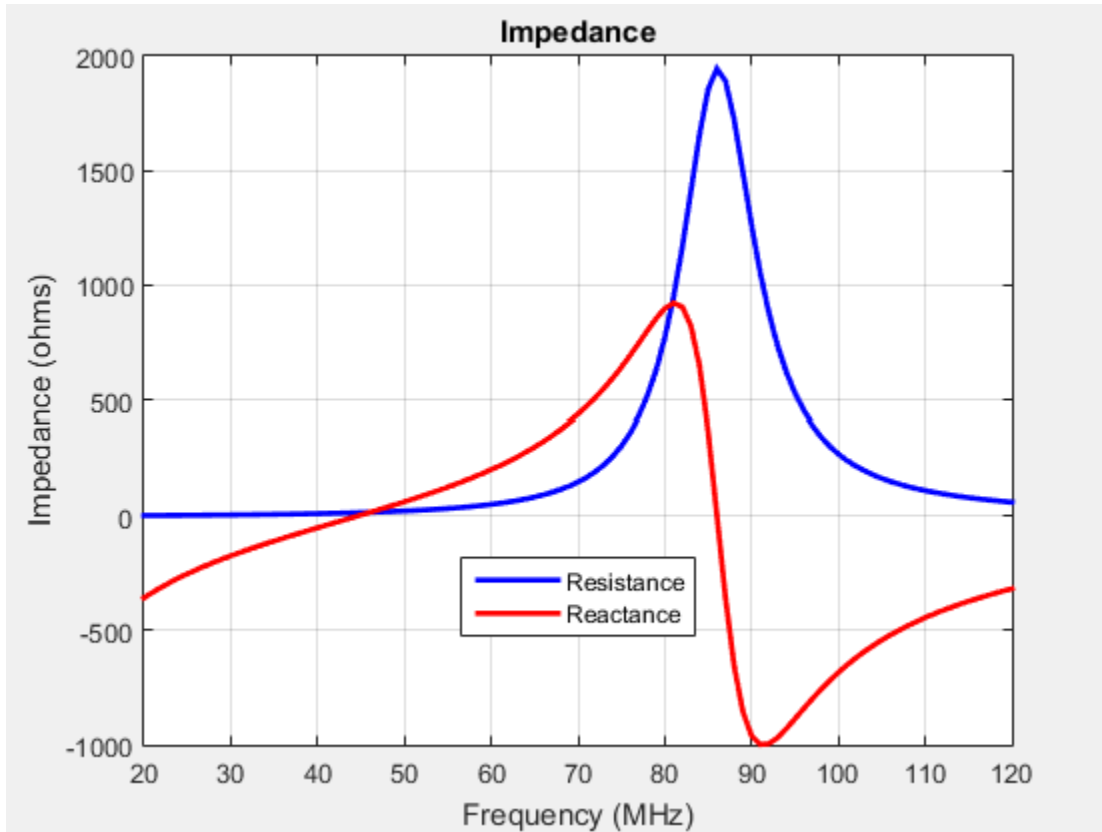

All the dimensions of this antenna are same as the monopole. The first resonance of the antenna is approximately 45 MHz. To view the accumulated charges on the top-hat monopole, use the `charge` function:
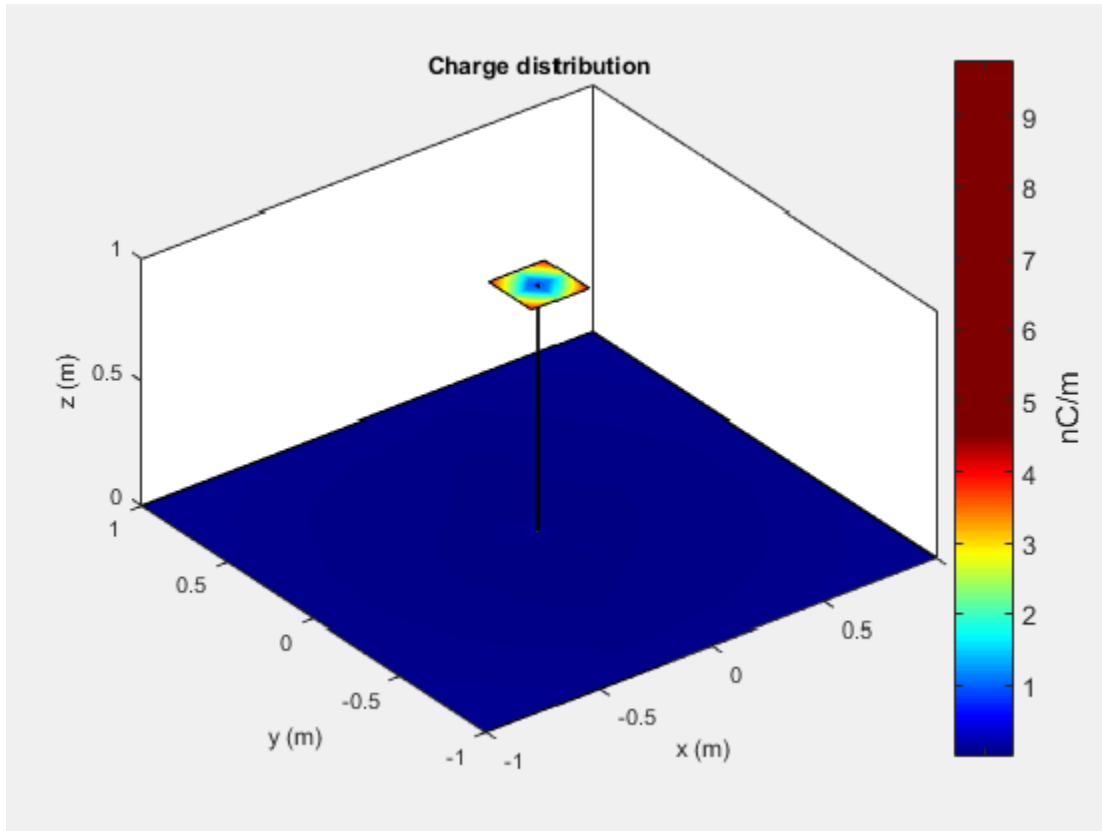
The increase in capacitance lowers the frequency of the antenna. By keeping the physical volume of the antenna same, the resonance point is shifted.

Increasing the top-hat dimensions provides more surface area for charges to accumulate. More charge accumulation increases the capacitance and pushes the resonant frequency lower. For example:

```
mt.TopHatLength = 0.35
mt.TopHatWidth = 0.35
impedance(mt,20e6:1e6:120e6)
```
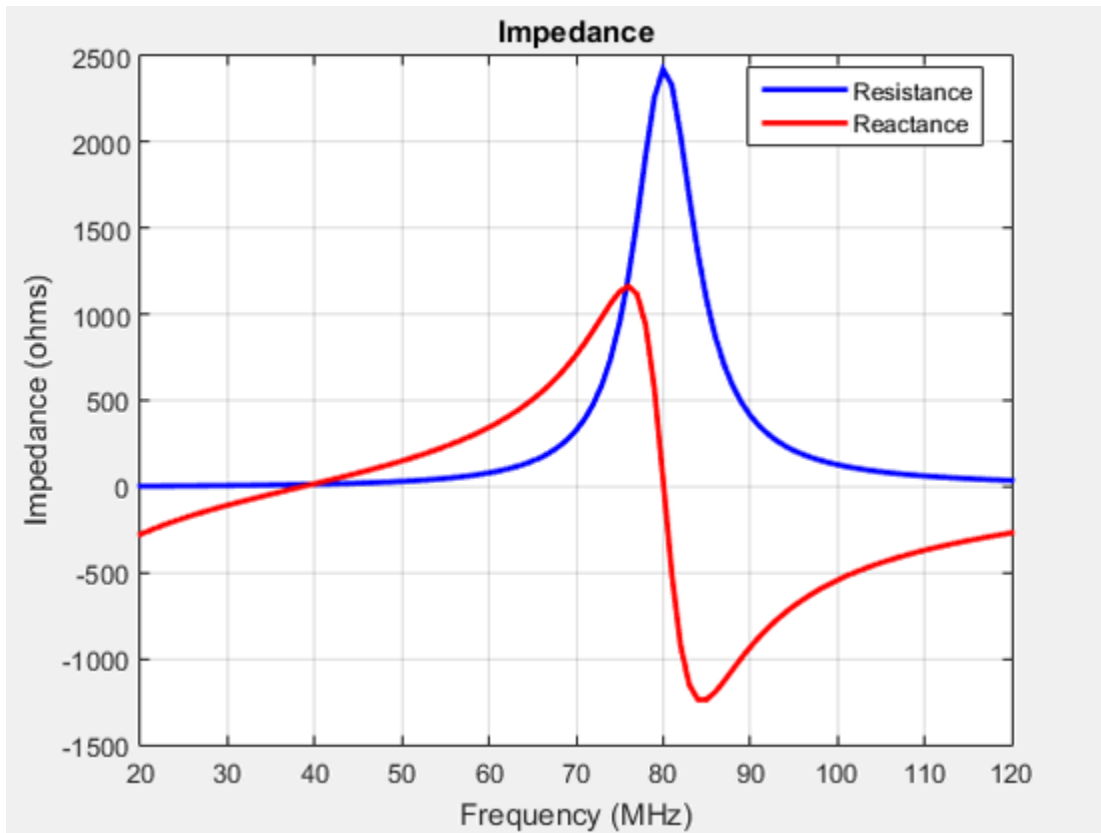
The resonance of the antenna further reduces to approximately 40 MHz.

Current Distribution

A typical antenna surface has current flowing on it. The behavior of the antenna surface current depends on the frequency of the input source, the geometry of the antenna, and the material properties of the antenna. The current is a vector and is spatially related to the structure of the antenna. In a dipole antenna, the maximum current distribution in the middle of the antenna and the minimum is toward the end:

```
d = dipole;
current(d,70e6)
```

1-15
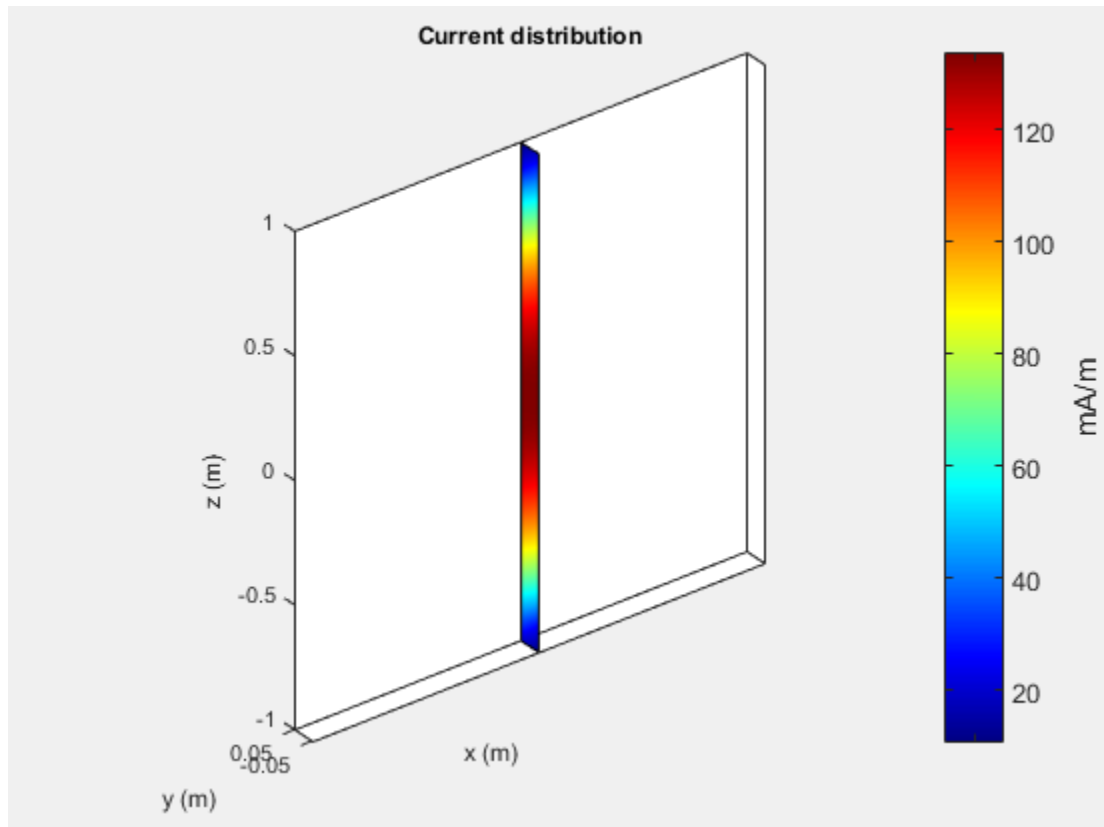
The same is true for a spiral antenna:

```
s = spiralEquiangular;
current(s,4e9)
```
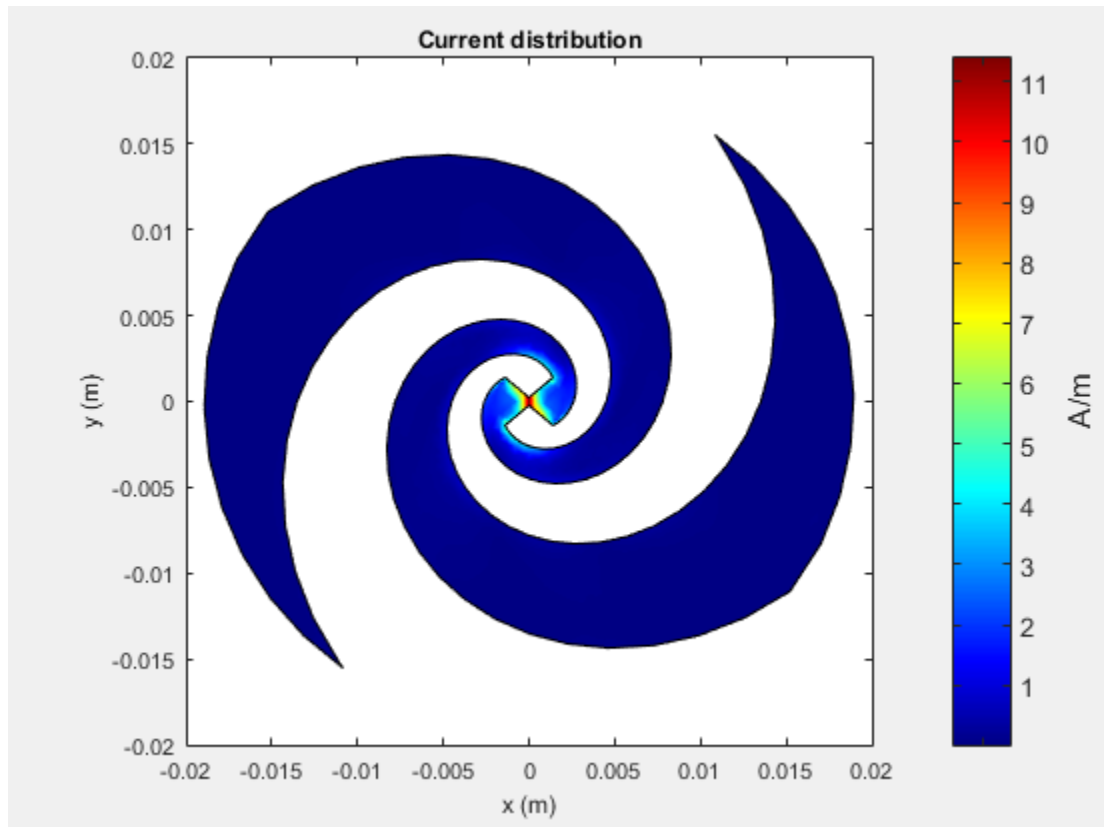
The patch also shows the current distribution of the classic $\lambda / 2$ open-open resistor. The two ends of the patch represent an open circuit since the current is at a minimum.

```
pm = patchMicrostrip;
current(pm,1.75e9)
```
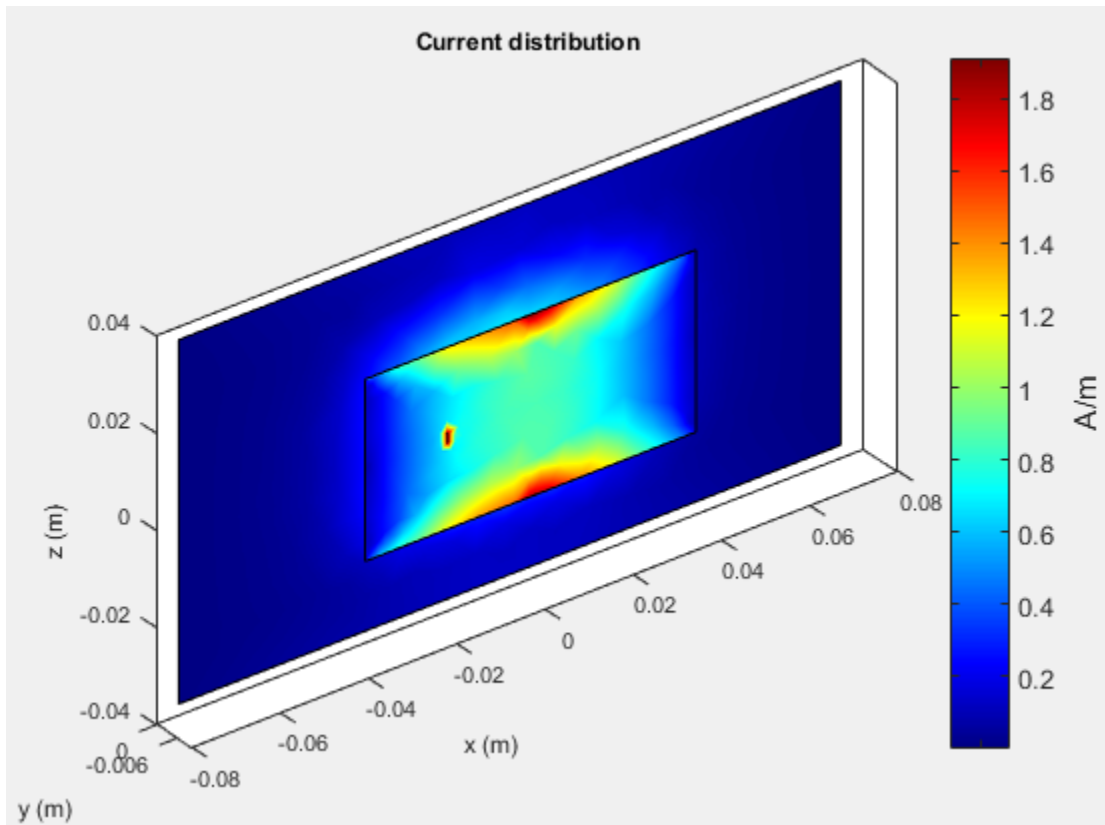
The spatial relationship between the current and structure of antenna is termed mode.

References

[1] Balanis, C.A. *Antenna Theory. Analysis and Design*, 3rd Ed. New York: Wiley, 2005.

[2] Makarov, S.N. *Antenna and EM Modeling with MATLAB*, New York: Wiley & Sons, 2002, p. 66.

Meshing

Method of Moments (MoM) is a numerical method that transforms Maxwell's continuous integral equations into an approximate discrete formulation that requires inversion of a large matrix. *Meshing* is the process of converting the continuous domain into the discrete domain for solving the equations. For discretizing surfaces, typically either triangles or rectangles are used. Antenna Toolbox uses triangular element for meshing as it conforms better for arbitrary shaped surfaces. The triangles are used to approximate the surface current using the Rao-Wilton-Glisson (RWG) basis functions. To get an accurate result, ensure that large number of triangles are present in the region where current variation is the highest. This region is typically either the corners in the antenna geometry or at the point where the antenna is excited.

In this section...
"Automatic Meshing Mode" on page 1-19
"Manual Meshing Mode" on page 1-21
"Strip Meshing" on page 1-22
"Surface Meshing" on page 1-22
"Meshing a Dielectric Substrate" on page 1-23

Automatic Meshing Mode

In Antenna Toolbox, the antenna structures mesh automatically based on the analysis frequency chosen. For analysis functions that accept a scalar frequency, the antennas mesh at that single frequency to satisfy the minimum triangles required. Then the functions calculate the corresponding antenna parameter.

```
d = dipole;
impedance(d,75e6);
mesh(d)
```
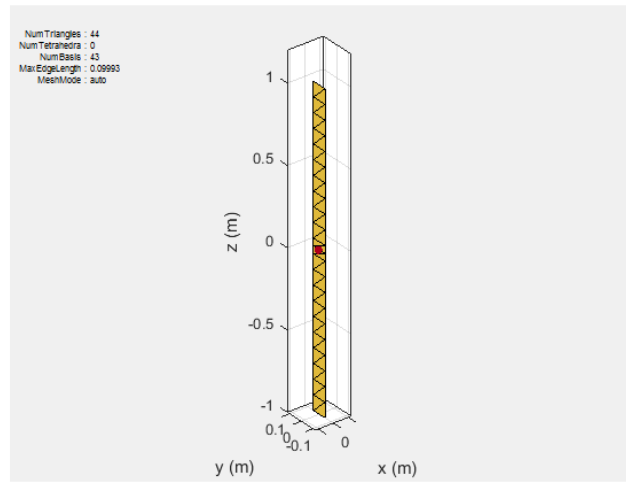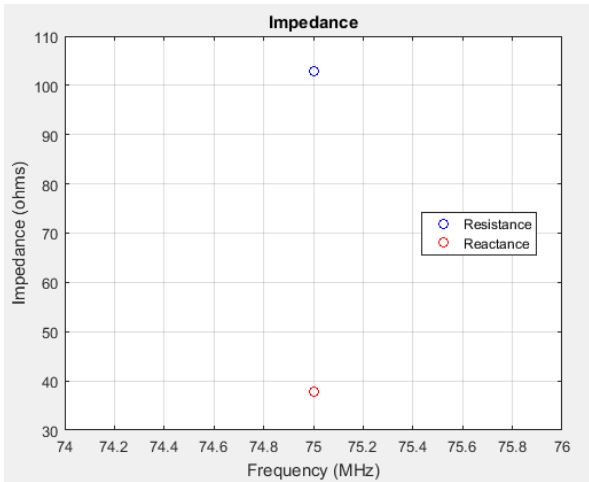
In above example, the dipole is meshed at 75 MHz automatically before calculating the impedance at that value. Use the mesh command to view the meshed dipole. The number of triangles is 44.

For analysis functions that accept a frequency vector (`impedance`, `sparameters`, `returnLoss`, `vswr`), each antenna meshes once at the highest frequency. Then, the functions calculate the corresponding antenna parameters at all the frequencies in the range.

```
d = dipole;
impedance(d,75e6:1e6;85e6);
mesh(d)
```
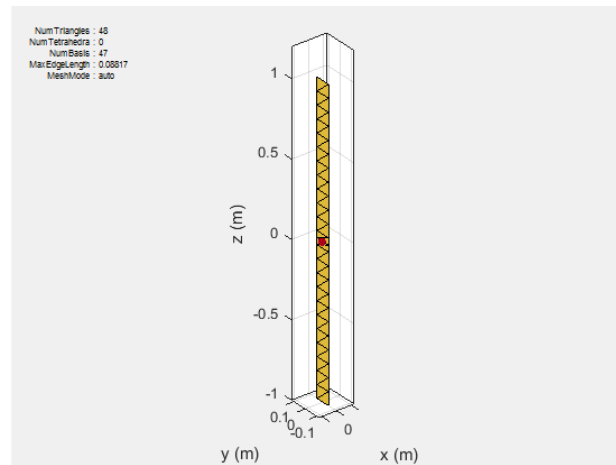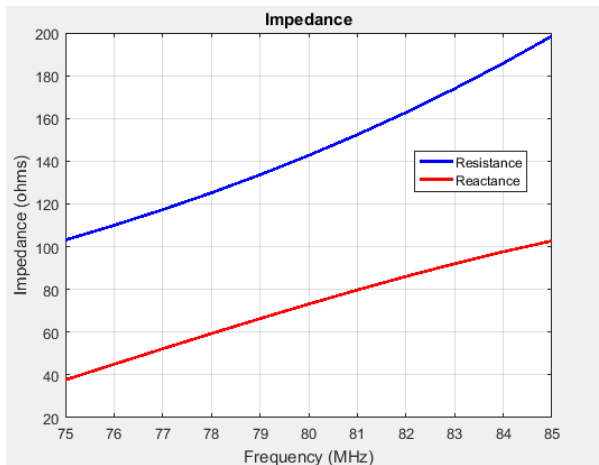
In above example, the dipole is meshed at the highest frequency, 85 MHz automatically before calculating the impedance at all the frequencies from 75 to 85 MHz. Meshing at the highest frequency, 85 MHz, ensures maximum number of triangles and a smoother plot of the dipole impedance. Use the mesh command to view the meshed dipole. The number of triangles is 48, which is more than single frequency meshing.

Manual Meshing Mode

You can choose to mesh the structure manually at the highest frequency of interest. Manual meshing is done by specifying the maximum edge length that is used for discretizing the structure. One option is to specify the value to be one-tenth of the wavelength at the highest frequency of interest. For example:

```
sp     = spiralArchimedean;
freq   = 0.8e9:100e6:2.5e9;
lambda = 3e8/freq(end);
mesh (sp,'MaxEdgeLength',lambda/10);
```

Alternatively, you can run an analysis at the highest frequency of interest and get the maximum edge length. Specify this maximum edge length using the mesh function as shown. This mesh is used for all other calculations.

```
sp     = spiralArchimedean;
freq   = 0.8e9:100e6:2.5e9;
temp   = axialRatio(sp,freq(end), 0, 90);
meshdata = mesh(sp);
```

```
mesh(sp,'MaxEdgeLength',meshdata.MaxEdgeLength);
```

Strip Meshing

For strip meshing, include at least 10 triangles per wavelength in a strip. This rule applies for structures such as dipoles, monopoles, and loops. Antenna Toolbox antenna meets the requirement automatically, based on the analysis frequency specified. The structured mesh generated in such cases is shown:
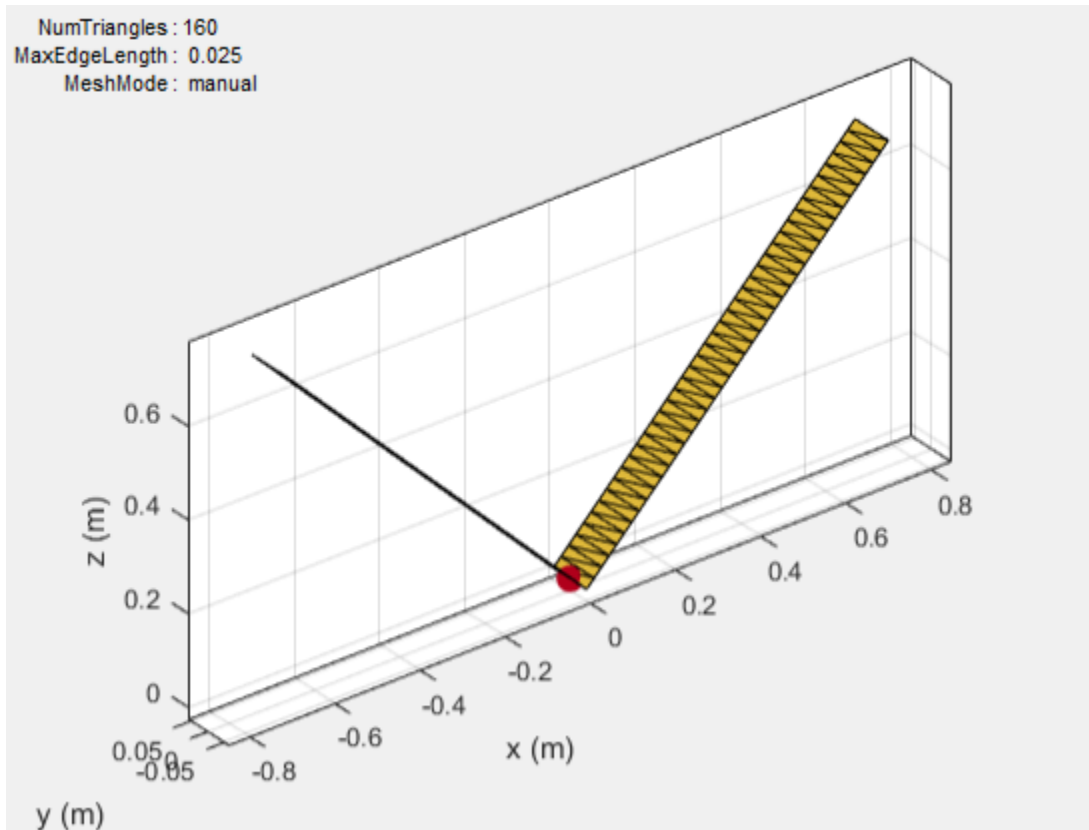
Surface Meshing

For surface meshing, it is recommended that there be at least 100 elements per wavelength in a particular area. This rule applies to structures such as spirals, patches,

and ground planes in general. Antenna Toolbox antenna meets the requirement automatically, based on the analysis frequency specified. In these cases, a non-uniform mesh is generated as shown:
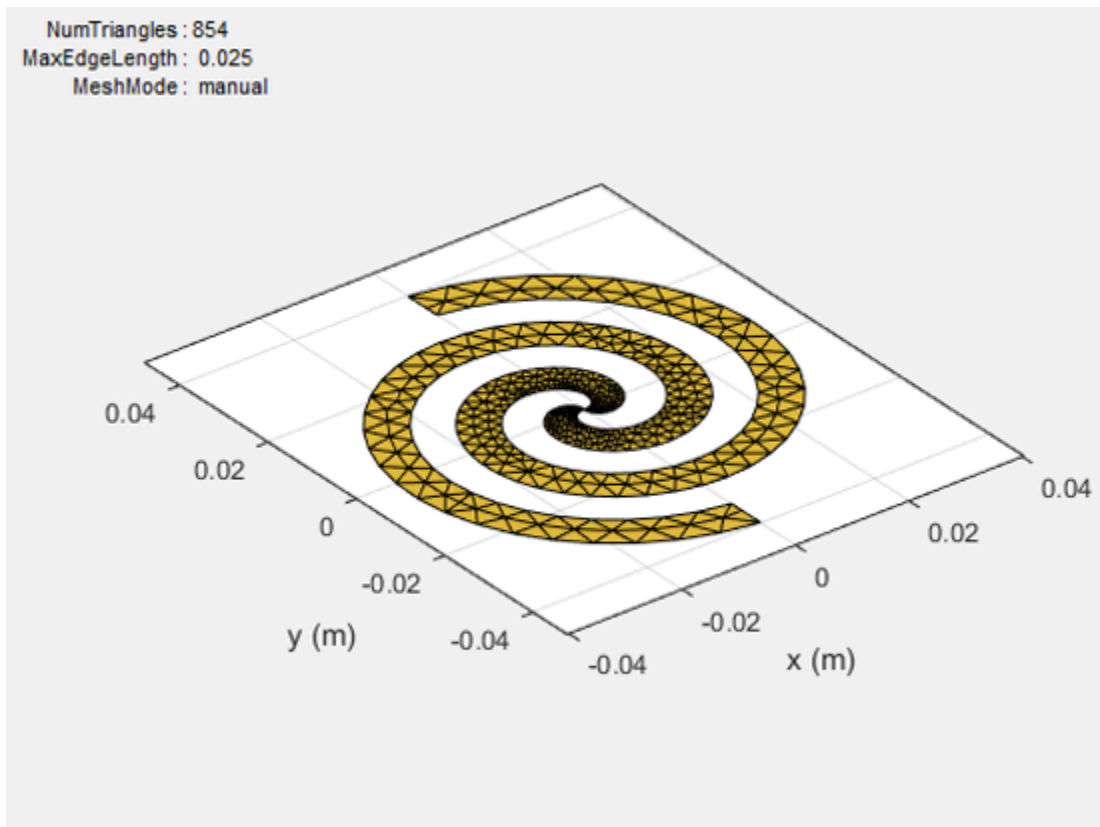
Larger number of triangles are added in regions with higher current density.

Meshing a Dielectric Substrate

For antennas using dielectrics and metals, Antenna Toolbox uses tetrahedrons to discretize the volume of the dielectric substrate.
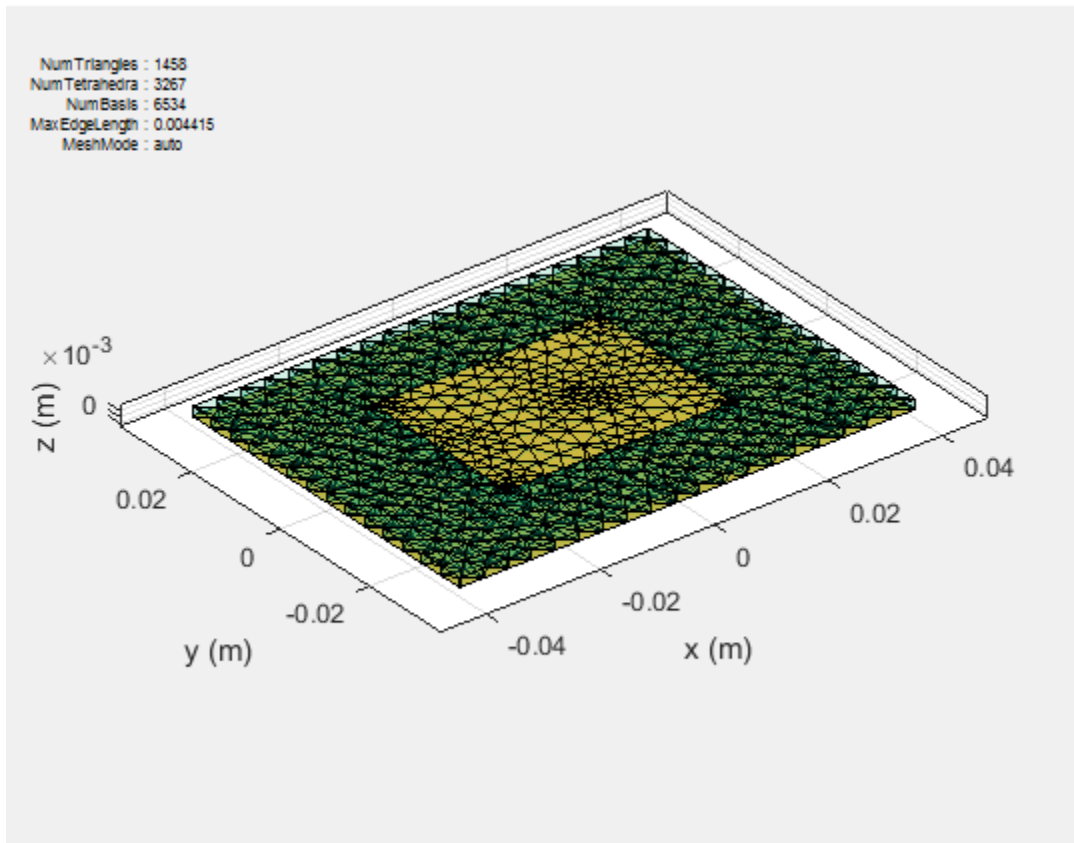
Thickness of the dielectric substrate is measured with respect to the wavelength. A dielectric substrate with thickness less than or equal to 1/50th of the wavelength is a thin substrate. When you mesh an antenna using dielectric in auto mode, thin substrates yield more accurate solutions.

A substrate with a thickness of 1/10th of the wavelength is a thick dielectric substrate. The method of moments solver requires 10 elements per wavelength to yield an accurate solution. Manual meshing yields more accurate solutions for antennas using thick dielectric substrate, as it satisfies the 10 elements per wavelength criteria.

References

[1] Makarov, S.N. *Antenna and EM Modeling with MATLAB*, New York: Wiley & Sons, 2002

Field Analysis

In this section...

Radiation Pattern

The *radiation pattern* of an antenna is the spatial distribution of power. The pattern displays the directivity or gain of the antenna. the *power pattern* of an antenna plots the transmitted or received power for a given radius. The *field pattern* of an antenna plots the variation in the electric or magnetic field for a given radius. The radiation pattern provides details such as the maximum and minimum value of the field quantity and the range of angles over which data is plotted.

```
h = helix;
h.Turns = 13;
h.Radius = 0.025;
pattern(h,2.1e9)
```

Output : Directivity
Frequency : 2.1 GHz
Max value : 14.1 dBi
Min value : -12.7 dBi
  Azimuth : [-180° , 180°]
Elevation : [-90° , 90°]

Main Lobe

Side Lobe

Null

Back Lobe
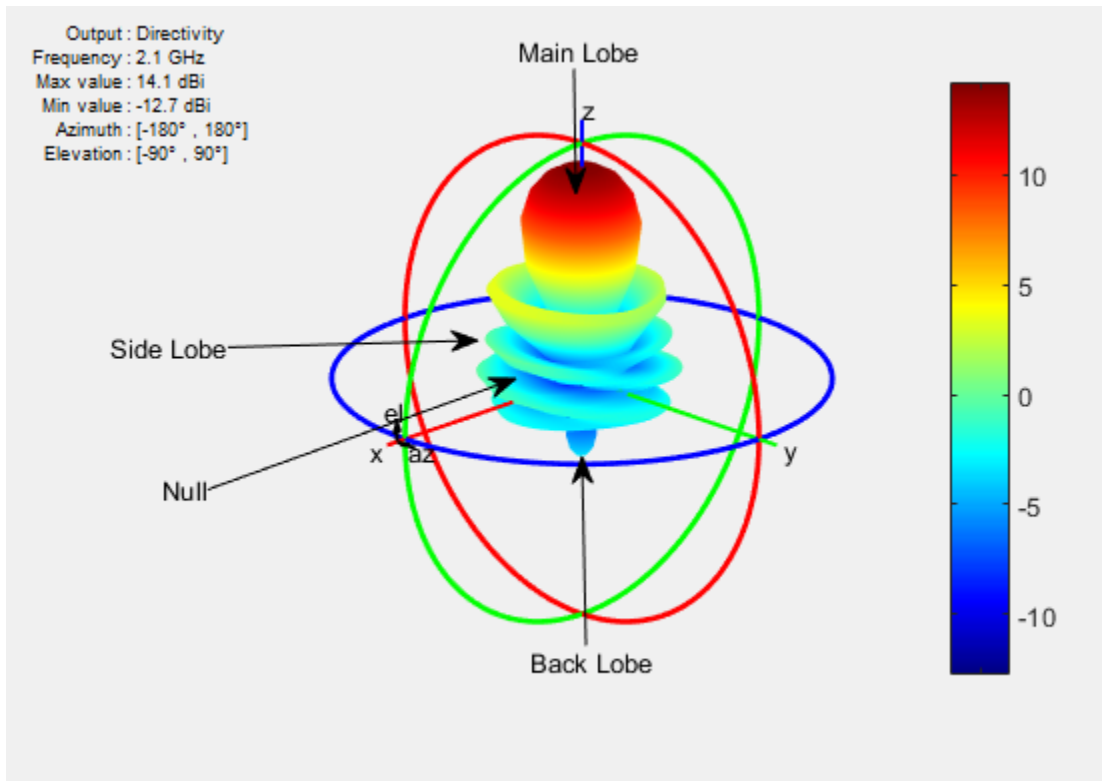
Use the `pattern` function to plot radiation pattern of any antenna in the Antenna Toolbox. By default, the function plots the directivity of the antenna. You can also plot the electric field and power pattern by using *Type* name-value pair argument of the pattern function.

Lobes

Each radiation pattern of an antenna contains *radiation lobes*. The lobes are divided into *major lobes* (also called main lobes) and *minor lobes*. *Side lobes* and *back lobes* are variations of minor lobes.

```
h = helix;
h.Turns = 13;
h.Radius = 0.025;
patternElevation(h,2.1e9)
```

Directivity (dBi) @ 2.10 GHz
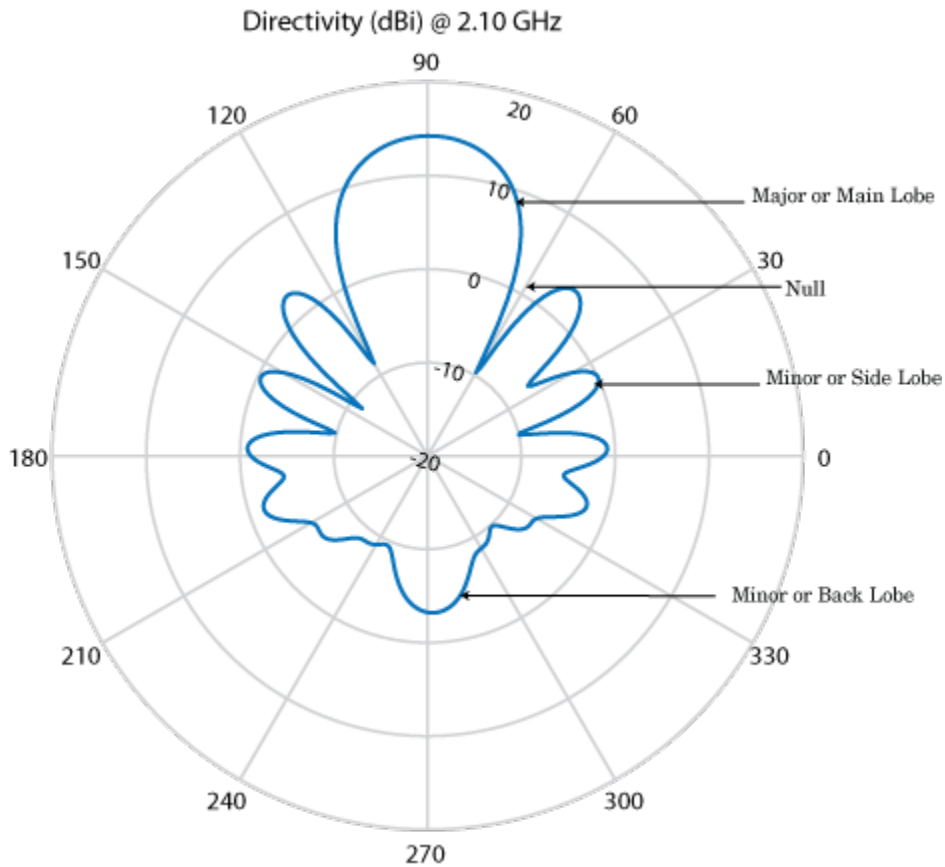
- **Major or Main lobe**: Shows the direction of maximum radiation, or power, of the antenna.
- **Minor lobe**: Shows the radiation in undesired directions of antenna. The fewer the number of minor lobes, the greater the efficiency of the antenna. *Side lobes* are minor lobes that lie next to the major lobe. *Back lobes* are minor lobes that lie opposite to the major lobe of antenna.
- **Null**: Shows the direction of zero radiation intensity of the antenna. Nulls usually lie between the major and minor lobe or in between the minor lobes of the antennas.

Field Regions

For an antenna engineer and an electromagnetic compatibility (EMC) engineer, it is important to understand the regions around the antenna.
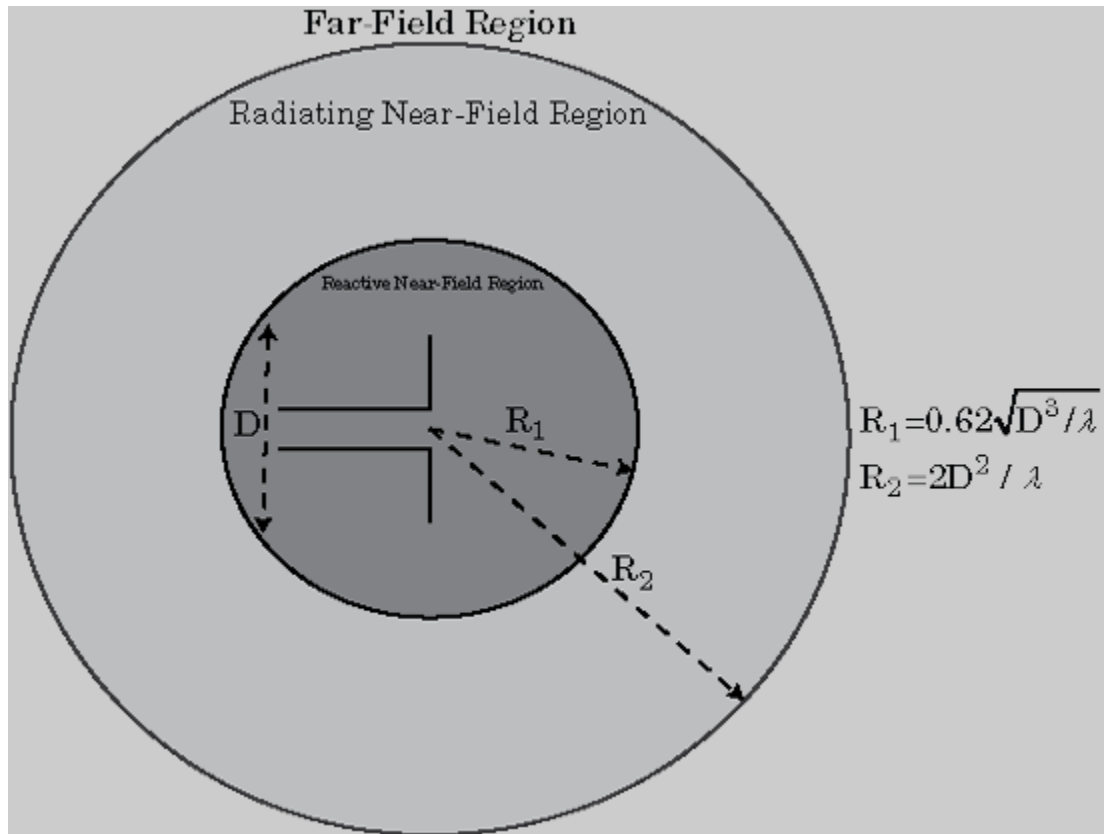
The region around an antenna is defined in many ways. The most used description is a 2- or 3-region model. The 2-region model uses the terms *near field* and the *far field* to identify specific dominant field mechanisms. The diagram is a representation of antenna fields and boundaries. The 3-field region splits the near field into a transition zone, where a weakly radiative mechanism is at work.

`Near-Field Region`: The *near-field region* is divided into two transition zones: a reactive zone and radiating zone.

- `Reactive Near-Field Region`: This region is closest to the antenna surface. The reactive field dominates this region. The reactive field is stored energy, or standing waves. The fields in this region change rapidly with distance from the antenna. The equation for outer boundary of this region is: $R < 0.62\sqrt{D^3/\lambda}$ where R is the distance from the antenna, λ is the wavelength, and D is the largest dimension of the antenna. This equation holds true for most antennas. In a very short dipole, the outer boundary of this region is $\lambda/2\pi$ from the antenna surface.

- `Radiating Near-Field Region`: This region is also called the *Fresnel region* and lies between the reactive near-field region and the far-field region. The existence of this region depends on the largest dimension of the antenna and the wavelength of operation. The radiating fields are dominant in this region. The equation for the inner boundary of the region is equation $R \geq 0.62\sqrt{D^3/\lambda}$ and the outer boundary is $R < 2D^2/\lambda$. This holds true for most antennas. The field distribution depends on the distance from the antenna.

`Far-field Region`: This region is also called *Fraunhofer region*. In this region, the field distribution does not depend on the distance from the antenna. The electric and magnetic fields in this region are orthogonal to each other. This region contains propagating waves. The equation for the inner boundary of the far-field is $R = 2D^2/\lambda$ and the equation for the outer boundary is infinity.

Directivity and Gain

Directivity is the ability of an antenna to radiate power in a particular direction. It can be defined as ratio of maximum radiation intensity in the desired direction to the average radiation intensity in all other directions. The equation for directivity is:

$$D = \frac{4\pi U(\theta,\phi)}{P_{rad}}$$

where:

- D is the directivity of the antenna
- U is the radiation intensity of the antenna
- P_{rad} is the average radiated power of antenna in all other directions

Antenna directivity is dimensionless and is calculated in decibels compared to the isotropic radiator (dBi).

The *gain* of an antenna depends on the directivity and efficiency of the antenna. It can be defined as the ratio of maximum radiation intensity in the desired direction to the total power input of the antenna. The equation for gain of an antenna is:

$$G = \frac{4\pi U(\theta, \phi)}{P_{in}}$$

where:

- G is the gain of the antenna
- U is the radiation intensity of the antenna
- P_{in} is the total power input to the antenna

If the efficiency of the antenna in the desired direction is 100%, then the total power input to the antenna is equal to the total power radiated by the antenna, that is, $P_{in} = P_{rad}$. In this case, the antenna directivity is equal to the antenna gain.

Beamwidth

Antenna beamwidth is the angular measure of the antenna pattern coverage. As seen in the figure, the main beam is a region around maximum radiation. This beam is also called the major lobe, or main lobe of the antenna.

Directivity (dBi) @ 2.10 GHz
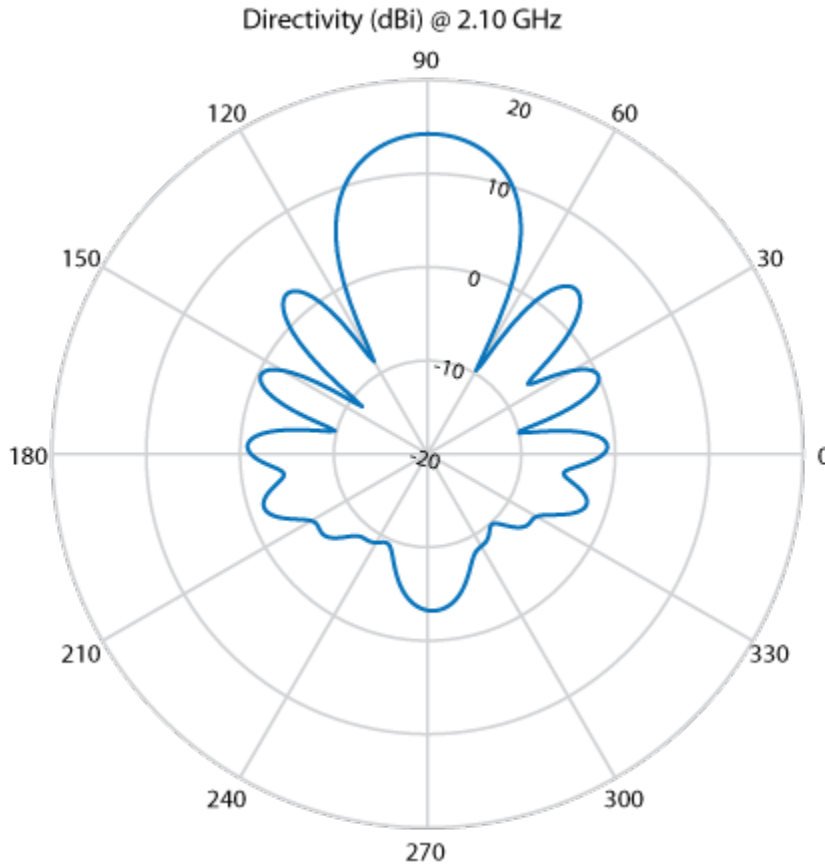
Half power beamwidth (HPBW) is the angular separation in which the magnitude of the radiation pattern decreases by 50% (or -3dB) from the peak of the main beam

Use the beamwidth function to calculate the beamwidth of any antenna in Antenna Toolbox.

E-Plane and H Plane

E-plane: Plane containing the electric field vector and the direction of maximum radiation. Consider a dipole antenna that is vertical along the z-axis. Use the patternElevation function to plot the elevation plane pattern. The elevation plane pattern shown captures the E-plane behavior of the dipole antenna.

```
d = dipole;
patternElevation(d,70e6)
```
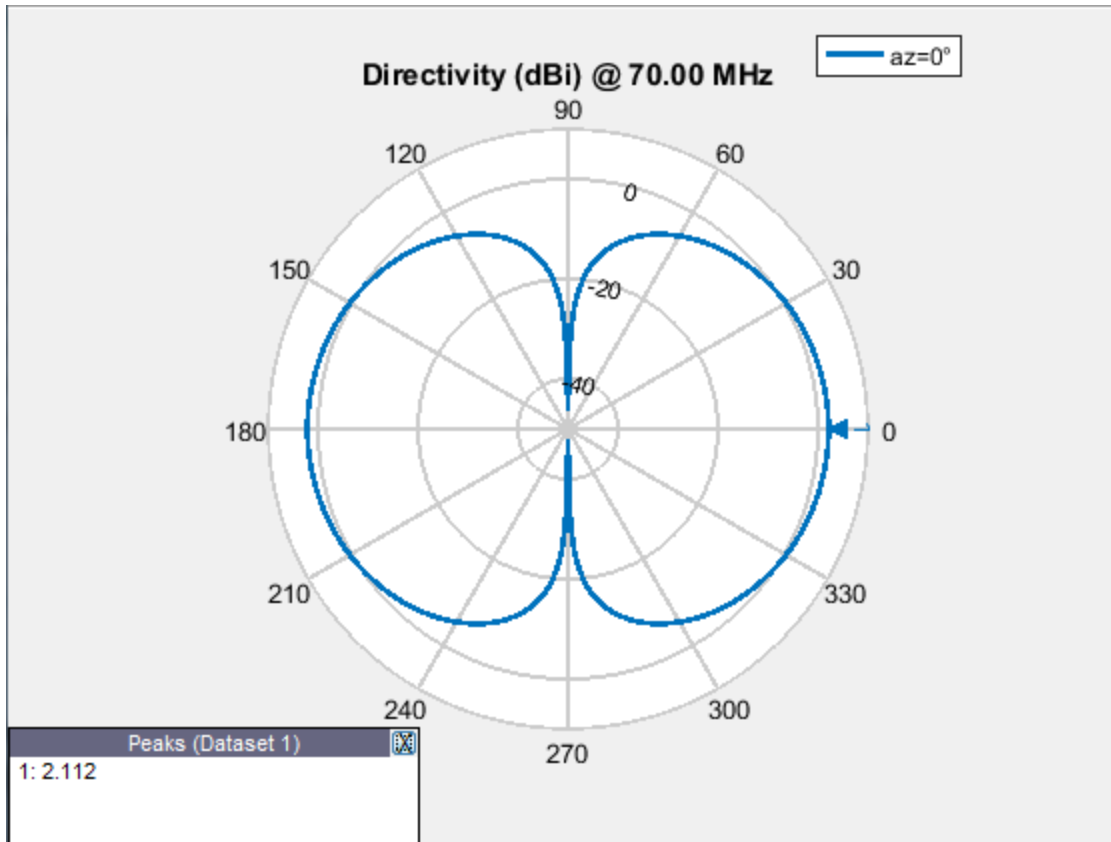

H-plane: Plane containing the magnetic field vector and the direction of maximum radiation. Use the `patternAzimuth` function to plot the azimuth plane pattern of a dipole antenna. The azimuthal variation in pattern shown captures the H-plane behavior of the dipole antenna.

```
d = dipole;
patternAzimuth(d,70e6)
```
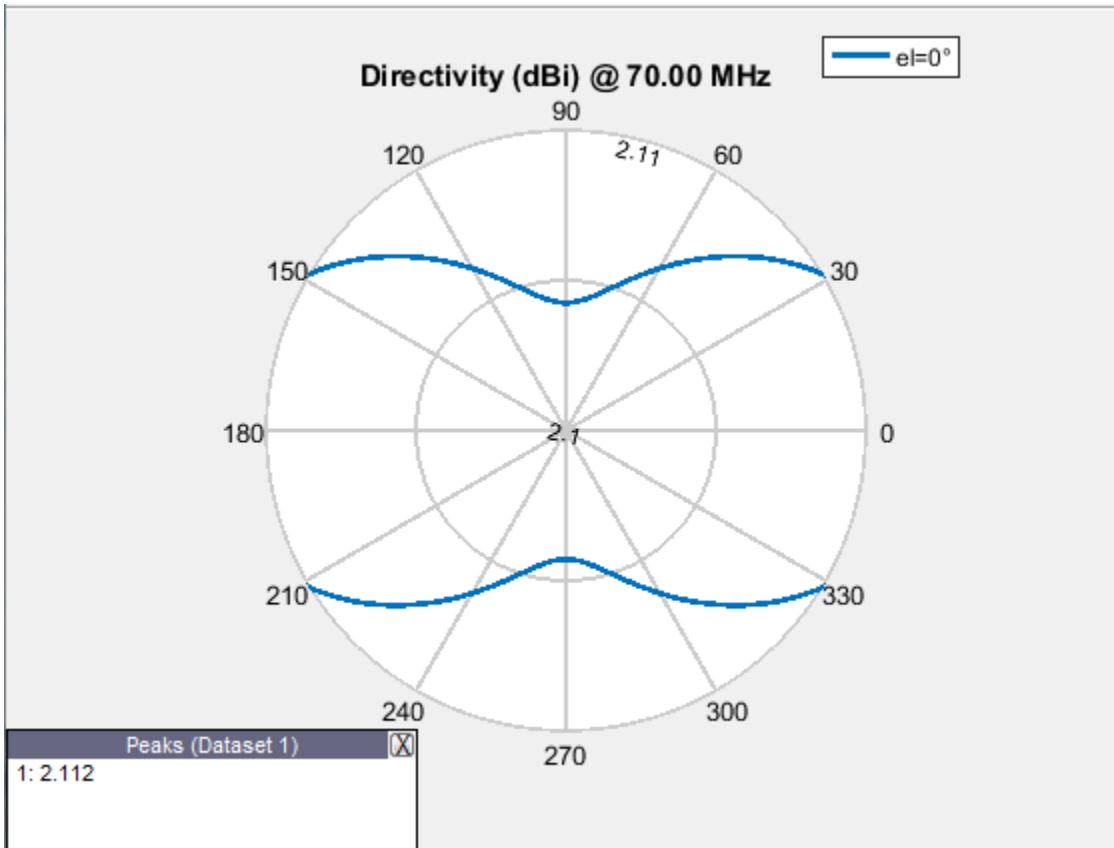
Use `EHfields` to measure the electric and magnetic fields of the antenna. The function can be used to calculate both near and far fields.

Polarization

Polarization is the orientation of the electric field, or `E-field`, of an antenna. Polarization is classified as elliptical, linear, or circular.

`Elliptical polarization`: If the electric field remains constant along the length but traces an ellipse as it moves forward, the field is elliptically polarized. Linear and circular polarizations are special cases of elliptical polarization.

`Linear polarization`: If the electric field vector at a point in space is directed along a straight line, the field is linearly polarized. A linearly polarized antenna radiates only one plane and this plane contains the direction of propagation of the radio waves. There are two types of linear polarization:

- `Horizontal Polarization`: The electric field vector is parallel to the ground plane. To view the horizontal polarization pattern of an antenna, use the `pattern` function, with the 'Polarization' name-value pair argument set to 'H'. The plot shows the horizontal polarization pattern of a dipole antenna:

```
d = dipole;
pattern(d,70e6,'Polarization','H')
```
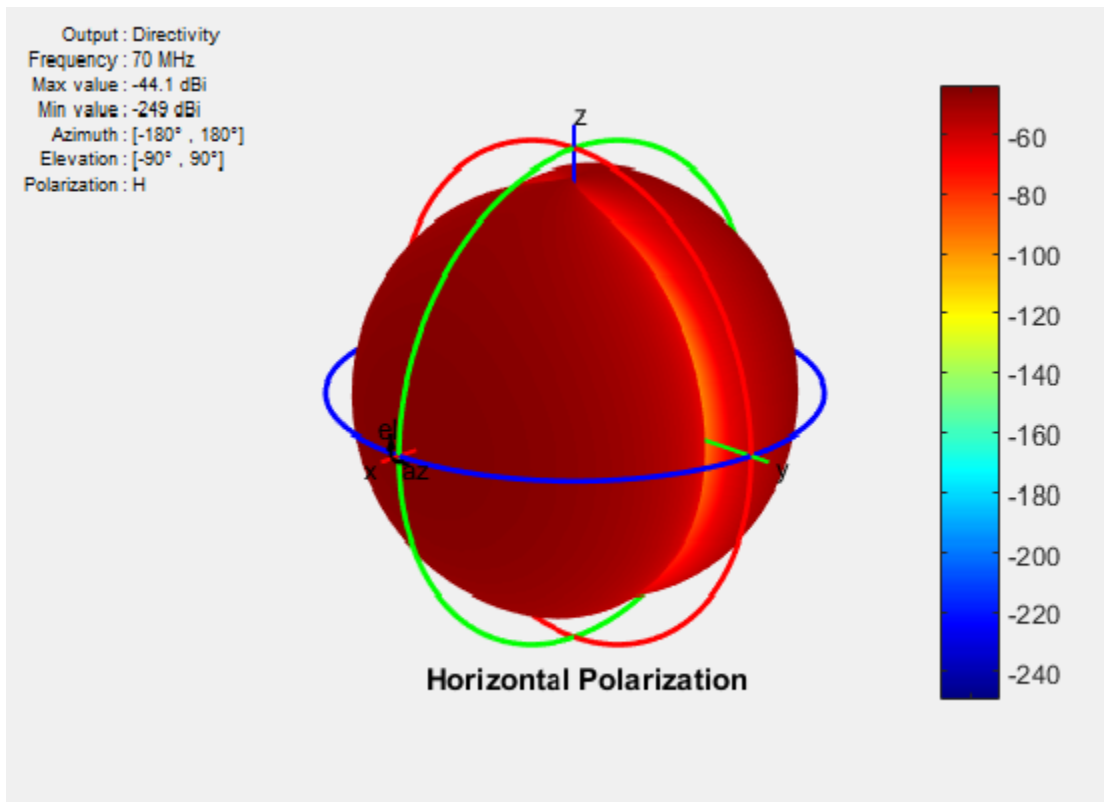

USA television networks use horizontally polarized antennas for broadcasting.

- `Vertical Polarization`: The electric field vector is perpendicular to the ground plane. To view the vertical polarization pattern of an antenna, use the `pattern` function, with the 'Polarization' name-value pair argument set to 'V'. Vertical polarization is used when a signal has to be radiated in all directions. The plot shows the vertical polarization pattern of a dipole antenna:

  ```
  d = dipole;
  pattern(d,70e6,'Polarization','V')
  ```
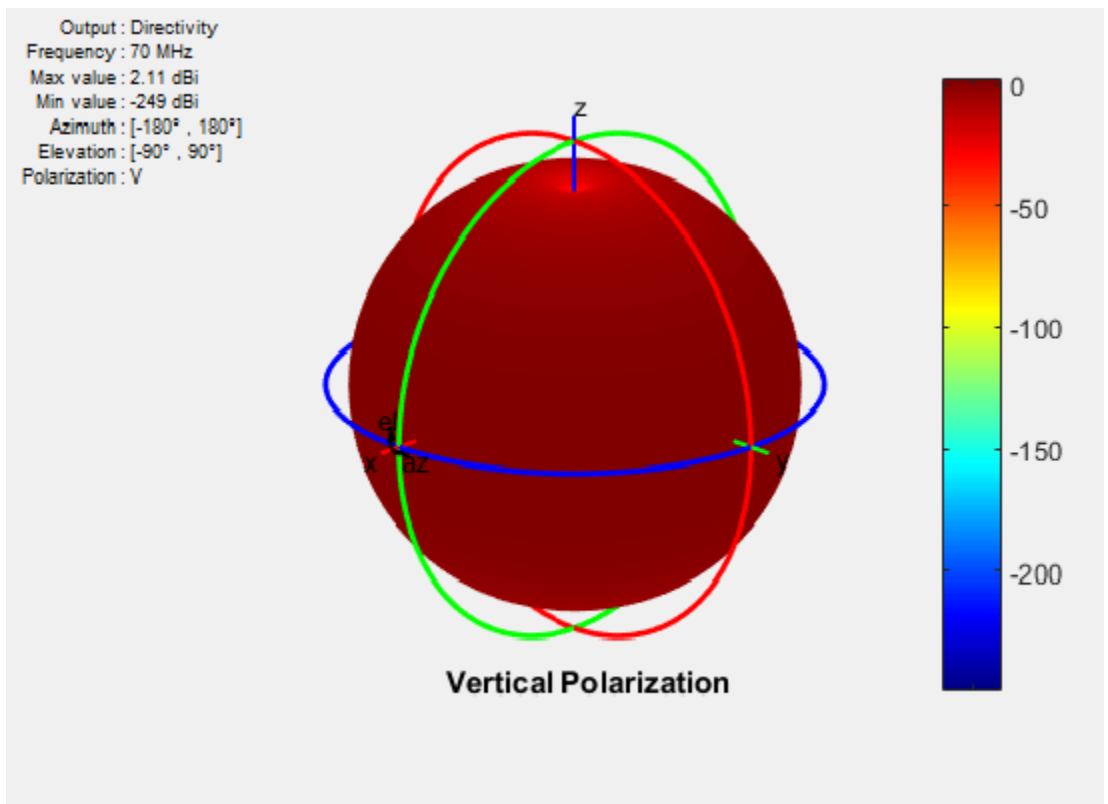

  An AM radio broadcast antenna or an automobile whip antenna are some examples of vertically polarized antennas.

`Circular Polarization`: If the electric field remains constant along the straight line but traces circle as it moves forward, the field is circularly polarized. This wave radiates

in both vertical and horizontal planes. Circular polarization is most often used in satellite communications. There are two types of circular polarization:

- **Right-Hand Circularly Polarized (RHCP)**: The electric field vector is traced in the counterclockwise direction. To view the RHCP pattern of an antenna, use the `pattern` function, with the 'Polarization' name-value pair argument set to 'RHCP'. The plot shows RHCP pattern of helix antenna:

```
h = helix;
h.Turns = 13;
h.Radius = 0.025;
pattern(h,1.8e9,'Polarization','RHCP')
```
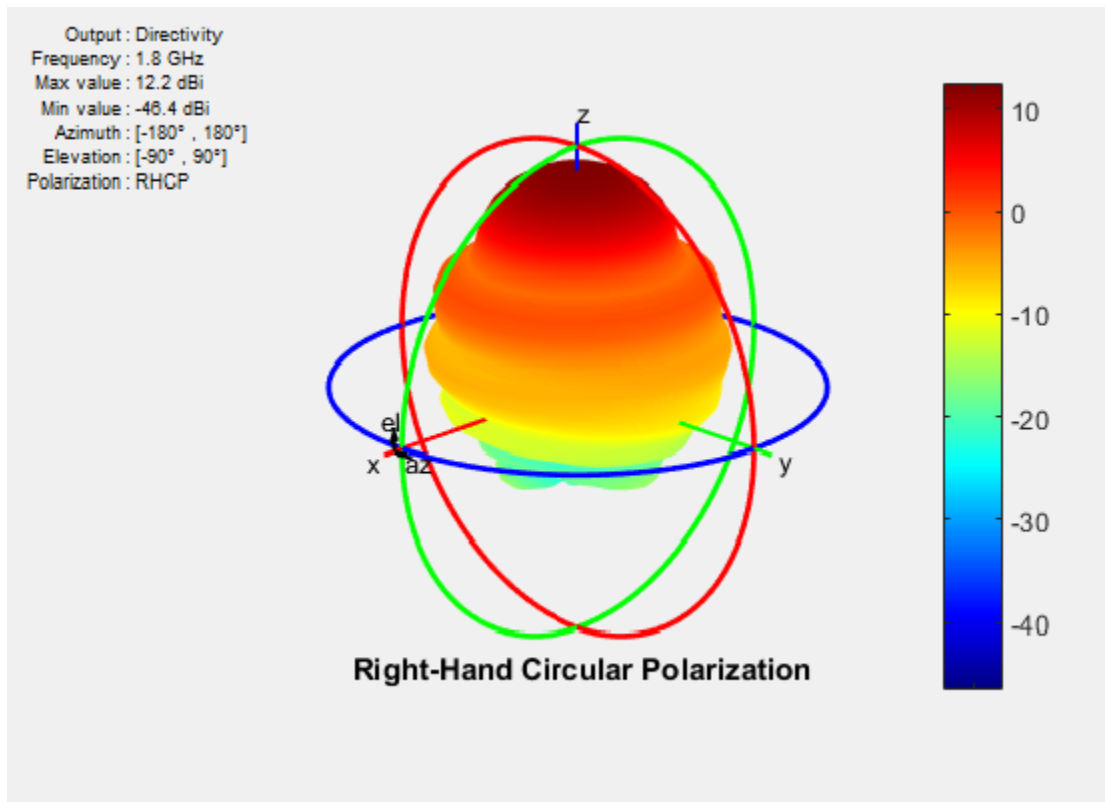

- **Left-Hand circularly polarized (LHCP)**: The electric field vector is traced in the clockwise direction. To view the LHCP pattern of an antenna, use the `pattern`

function, with the 'Polarization' name-value pair argument set to 'LHCP'. The plot shows LHCP pattern of helix antenna:

```
h = helix;
h.Turns = 13;
h.Radius = 0.025;
pattern(h,1.8e9,'Polarization','LHCP')
```
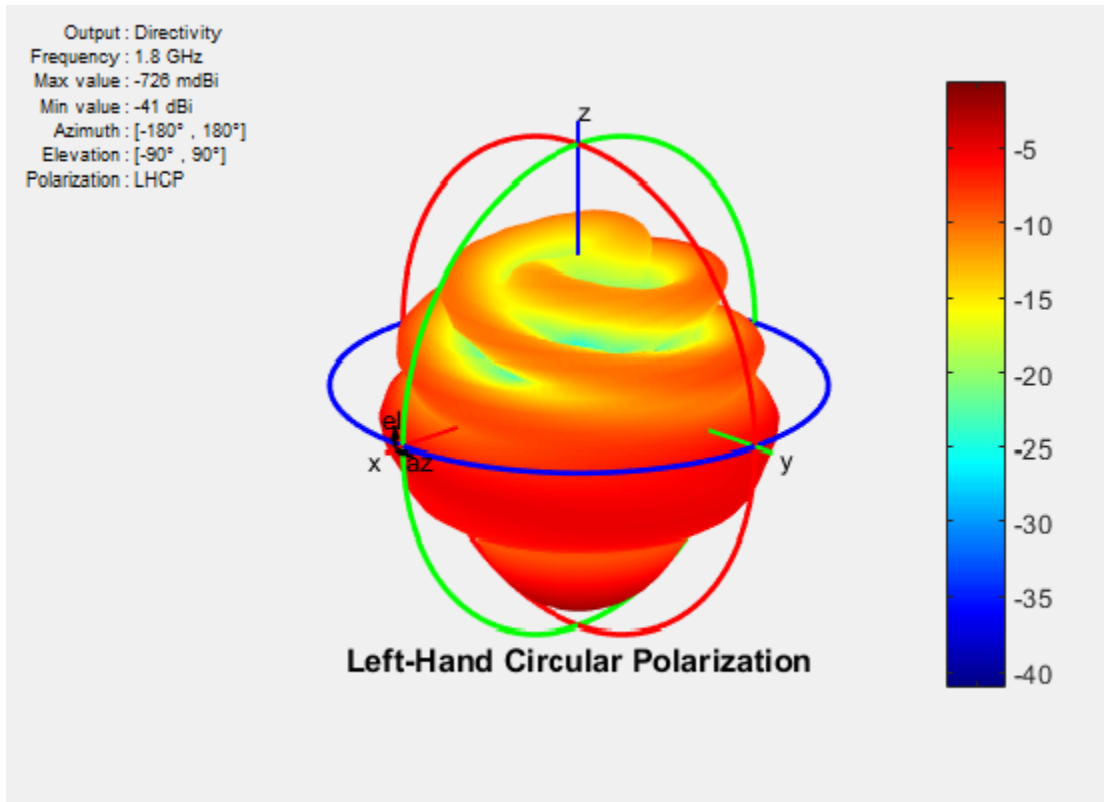

For efficient communications, the antennas at the transmitting and receiving end must have same polarization.

Axial Ratio

Axial ratio (AR) of an antenna in a given direction quantifies the ratio of orthogonal field components radiated in a circularly polarized wave. An axial ratio of infinity implies a linearly polarized wave. When the axial ratio is 1, the radiated wave has pure circular polarization. Values greater than 1 imply elliptically polarized waves.

Use `axialRatio` to calculate the axial ratio for any antenna in the Antenna Toolbox.

References

[1] Balanis, C.A. *Antenna Theory. Analysis and Design*, 3rd Ed. New York: Wiley, 2005.

[2] Stutzman, Warren.L and Thiele, Gary A. *Antenna Theory and Design*, 3rd Ed. New York: Wiley, 2013.

[3] Capps, C. *Near Field or Far Field*, EDN, August 16, 2001, pp.95 - pp.102.

Rotate Antenna and Arrays

In this section...
"Rotate Single Antenna Element " on page 1-40
"Rotate Single Antenna Element About Multiple Axis" on page 1-43
"Rotate Dipole Backed Reflector" on page 1-44
"Rotate Antenna Array Elements" on page 1-47

To rotate antenna elements in Antenna Toolbox, use the `Tilt` and `TiltAxis` property. The `Tilt` property is a scalar or a vector and specifies the angle of rotation of the antenna in degrees. The `TiltAxis` property is a vector and specifies the axis of rotation (*x*, *y*, *z*]) of the antenna. Antenna arrays do not have these properties and cannot be rotated.

Rotate Single Antenna Element

Create a patch antenna. By default, this antenna is on the X-Y plane.

```
patch = patchMicrostrip;
show(patch)
```
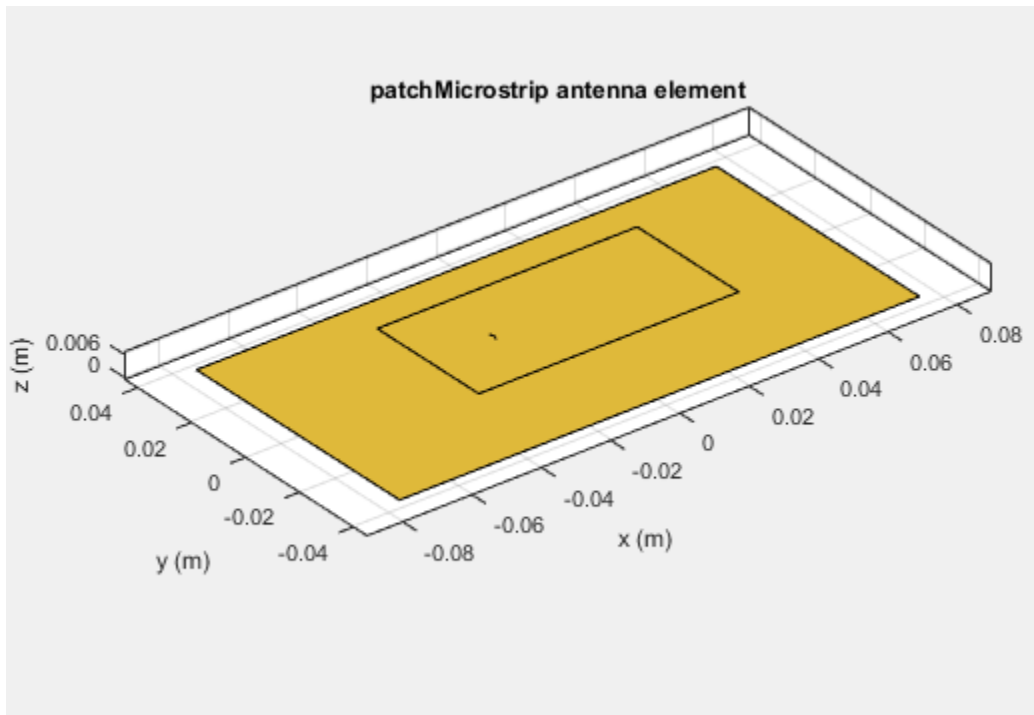
To make the antenna lie on the Y-Z plane, rotate it by 90 degrees about the Y-axis. The rotation follows the standard right-hand rule.

```
patch.TiltAxis ='Y';
patch.Tilt = 90;
show(patch)
```
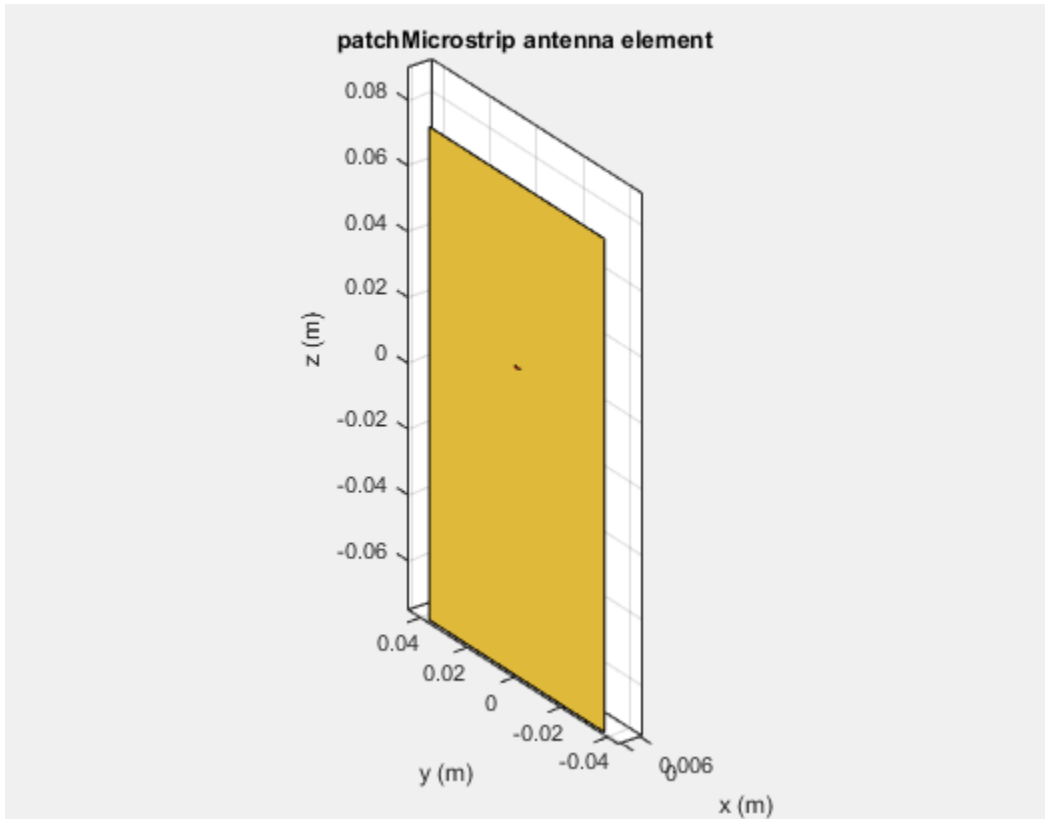
The patch lies behind the ground plane and is not visible. To make the patch visible (in front of the ground plane), rotate the antenna about the negative y-axis.

```
patch.TiltAxis = [0 -1 0];
show(patch)
```
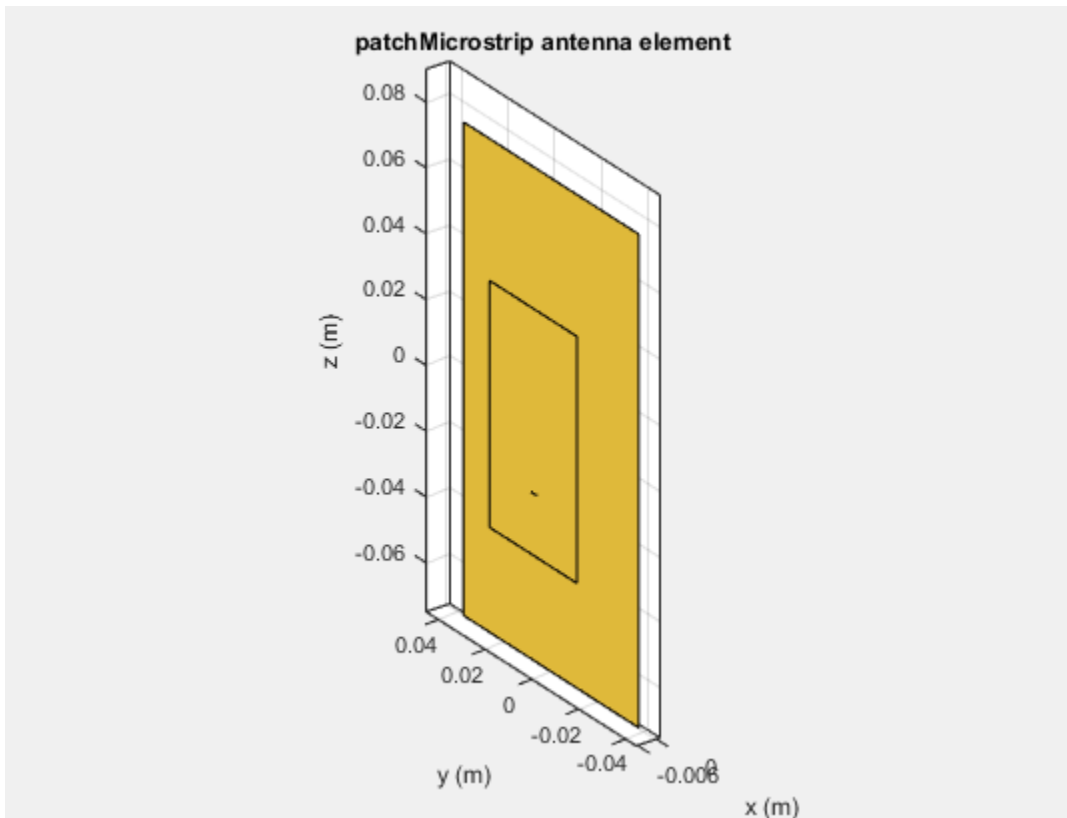
Rotate Single Antenna Element About Multiple Axis

Create a dipole antenna rotated at 90 degrees about two three-element vector points in space.

```
d = dipole('Tilt',[90 90],'TiltAxis', [0 1 0; 0 1 1])

dipole with properties:

      Length: 2
       Width: 0.1000
   FeedOffset: 0
        Tilt: [90 90]
     TiltAxis: [2x3 double]
```

```
show(d)
```
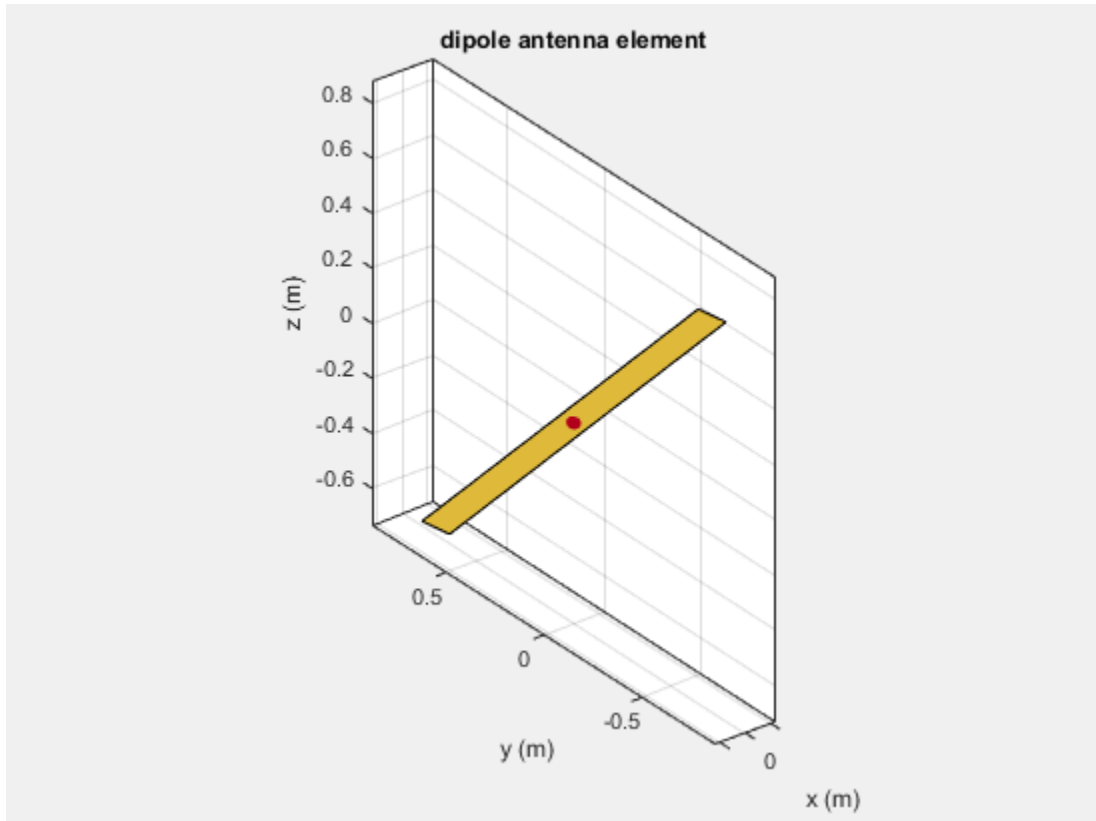

Rotate Dipole Backed Reflector

The `Tilt` and `TiltAxis` properties are available for dipole and reflector elements. Use these properties if you want to model the antenna parallel or perpendicular to the ground plane.

Create a reflector element. By default, the dipole is parallel to the reflector element.

```
r = reflector;
show(r)
```
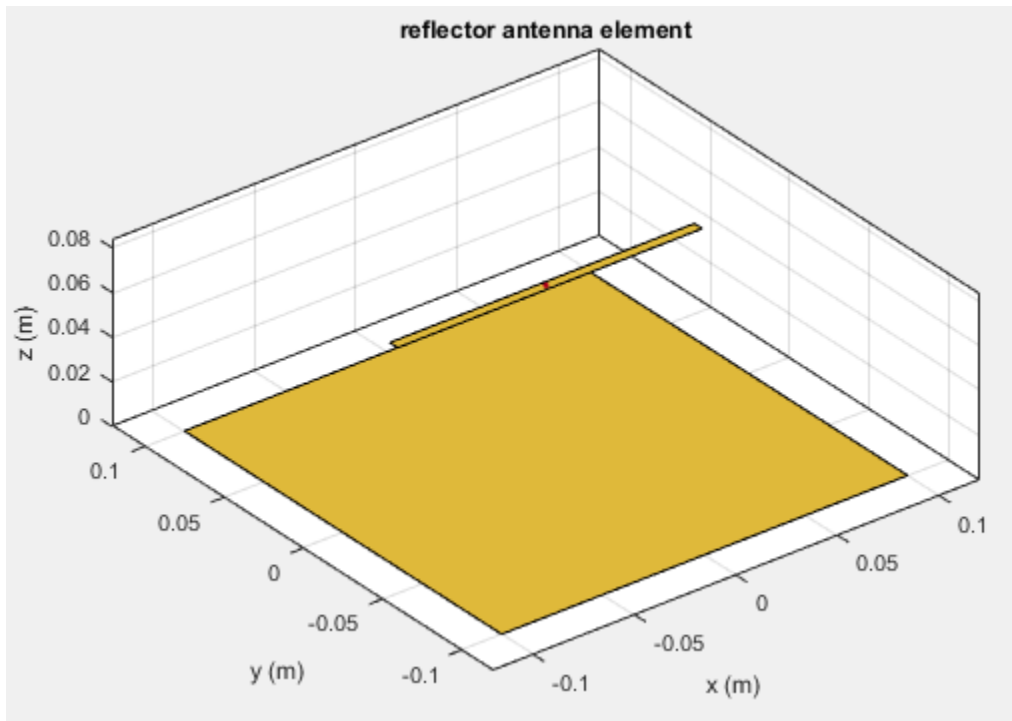
The dipole is the exciter element of the reflector. View its properties.

```
r.Exciter;
```

```
dipole with properties:

        Length: 0.1500
         Width: 0.0050
    FeedOffset: 0
          Tilt: 90
      TiltAxis: [0 1 0]
```

In the properties, you see that the dipole element is already rotated by 90 degrees about the y-axis and is parallel to the X-Y plane. To make the dipole perpendicular to the X-Y plane, change the tilt of the dipole to zero degrees.

```
r.Exciter.Tilt = 0;
show (r)
```

reflector antenna element
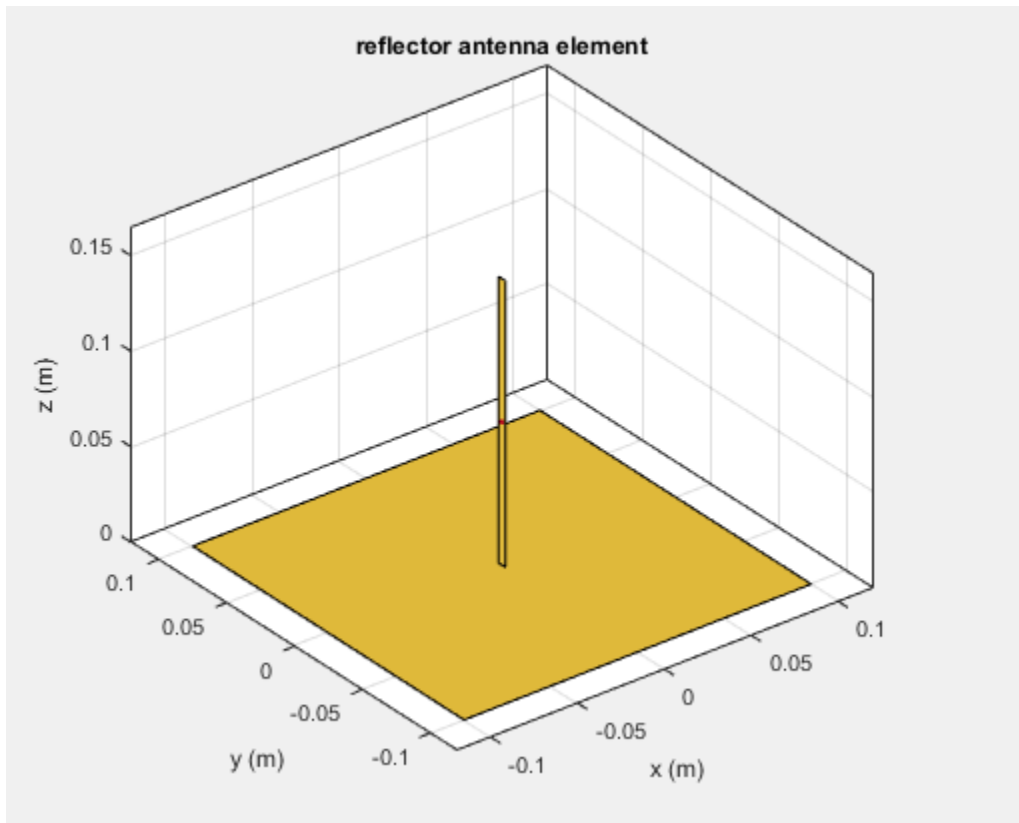
Rotating the reflector element rotates the entire structure by the specified angle. To rotate the complete antenna, you use the *Tilt* property. For example, rotate the reflector by 90 degrees about the y-axis. According to the right-hand rule, the reflector now lies in the Y-Z plane with the dipole.

```
r = reflector;
r.Tilt = 90;
show(r)
```

reflector antenna element
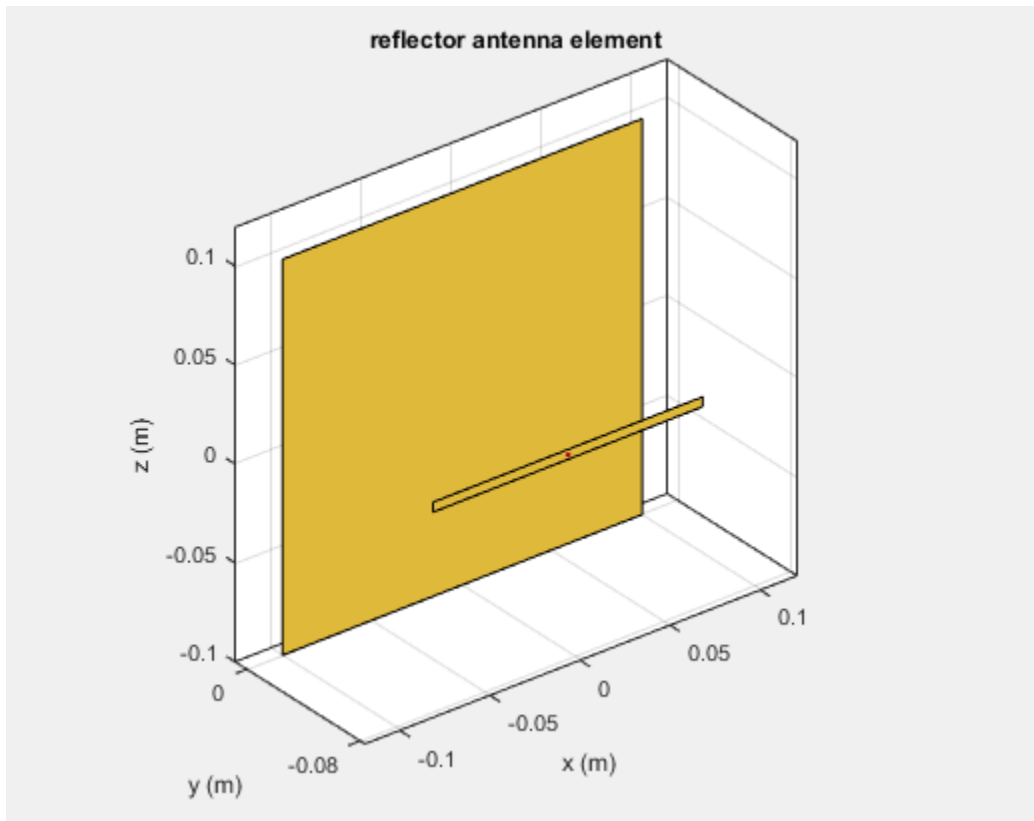
Rotate Antenna Array Elements

The `Tilt` and `TiltAxis` properties are available for antenna array elements.

Create an array of Archimedean spirals with each spiral rotated at 30 degrees about the x-axis and y-axis. First create one spiral antenna.

```
s = spiralArchimedean;
l = linearArray('Element',s,'ElementSpacing',0.1,'NumElements',3,'Tilt',...
30,'TiltAxis',[1 1 0]);
show(l)
```

linearArray of spiralArchimedean antennas
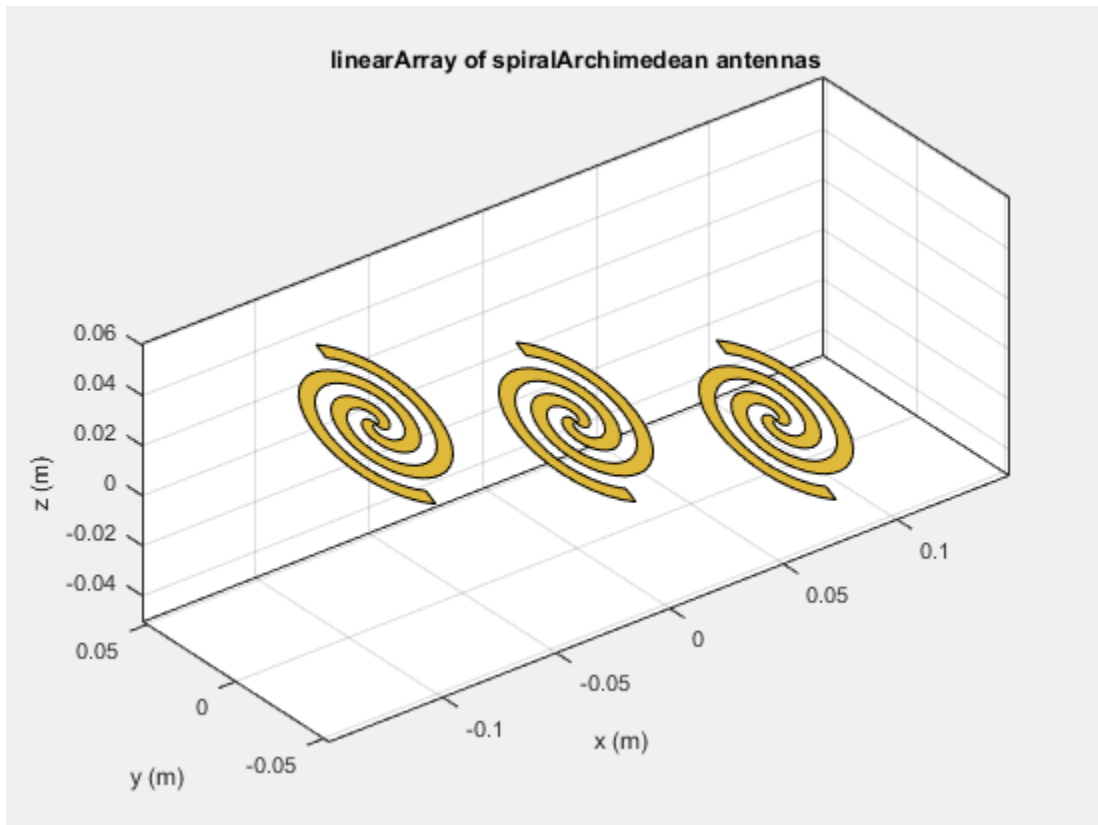
More About

· "Antenna Classification"

Infinite Ground Plane

In this section...
"Image Theory" on page 1-49
"Formulate the Image Theory Technique" on page 1-50

Antenna Toolbox library element uses the image theory technique to model an infinite ground plane. The main advantage of image theory technique is that you do not need to discretize the ground plane. Image theory reduces the overall size of the problem and you get a solution faster.

Image Theory

The image theory technique uses a catalog of electromagnetic problems that produce identical field distributions. You can identify these electromagnetic problems by noting that conducting surfaces are of constant potential. Placing these conducting surfaces along any equipotential lines in any field distributions does not alter the fields.

Consider a positive and negative charge placed a distance of *2x* apart from each other. The equipotential surface for these two charges forms at a distance midway between them.
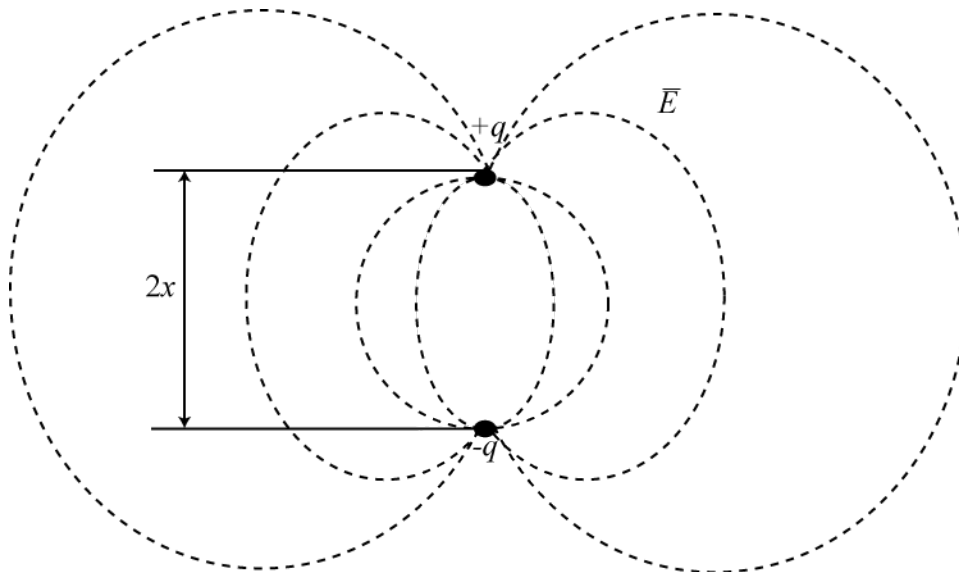
If you place a conducting object along this equipotential surface, then the field above the surface does not change.
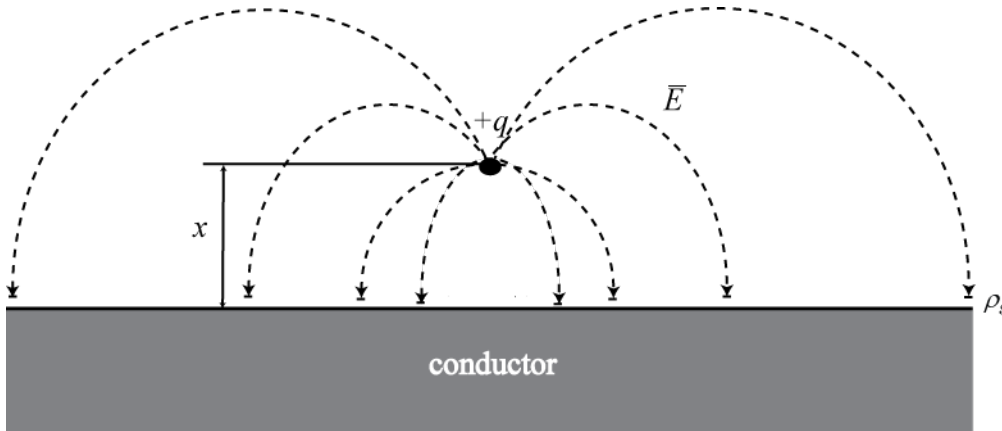
You can also apply this equivalence in reverse. Consider charges that are distance x apart from a conducting surface. You can replace the conducting surface with a set of image charges whose signs are opposite those of the original charge. Place these image charges at a distance x below the original conducting surface. This method eliminates the conducting plate, leaving only charges in unbound space. This equivalence is called the *method of images*.

Formulate the Image Theory Technique

Consider a rectangular plate over an infinite ground plane. This rectangular plate is divided into a triangular mesh. The induced currents flow over the triangulated surface using the RWG basis functions.

Original Problem:
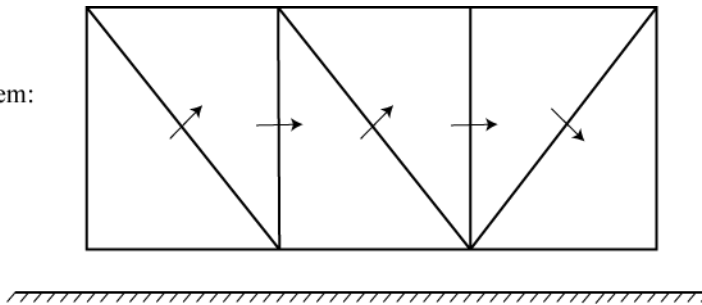
You formulate the problem by considering real and image basis functions. You can show the same problem using image theory or method of images. The interaction matrix is calculated using the interaction between real and image basis functions.

Equivalent Problem:
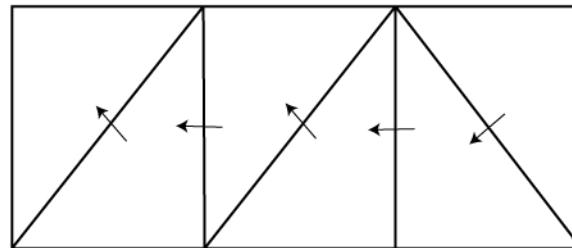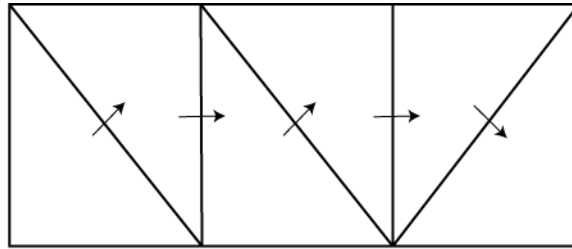
According to the method of images, the real and image electric currents are equal to each other in terms of magnitude. The parallel components are in opposite direction.The normal components are in the same direction.

Original Problem:

Equivalent Problem:
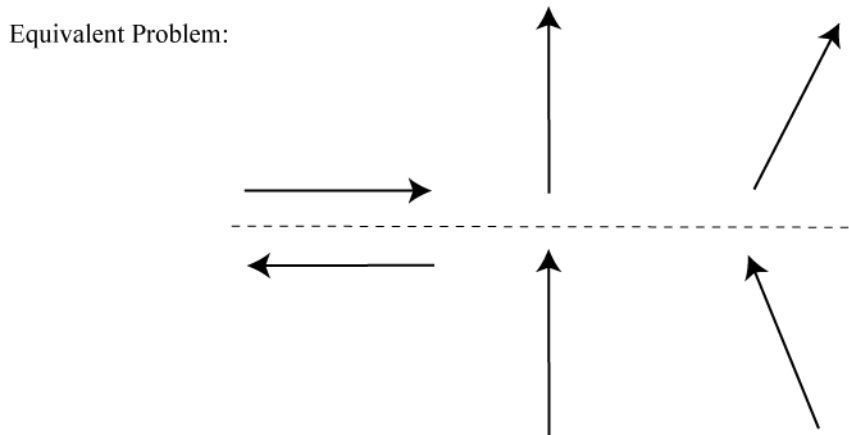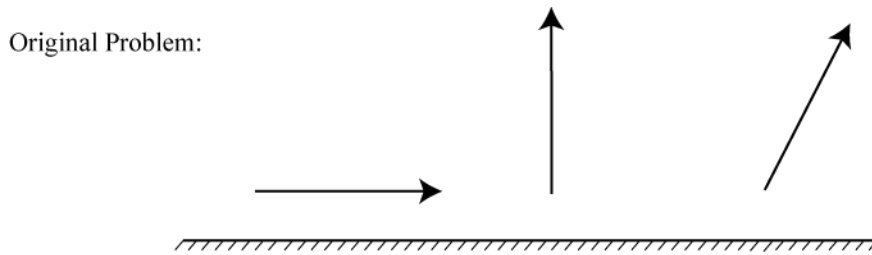
References

[1] Balanis, C.A. *Antenna Theory. Analysis and Design*, 3rd Ed. New York: Wiley, 2005.

[2] Yildrium, Cemal. "Scattering and Radiation problems of arbitrary shaped conducting bodies above ground plane" Ph.D. Thesis. Bilkent University, Ankara, Turkey, October, 2002

More About

- "Method of Moments – Metal" on page 3-2

Model Infinite Ground Plane for Balanced Antennas

This example shows how to model an infinite ground plane and calculate fundamental antenna parameters for balanced antennas.

Create Antenna On Infinite Ground Plane

Create a reflector antenna on a ground plane of infinite length.

```
r = reflector ('GroundPlaneLength', inf)


r =

  reflector with properties:

              Exciter: [1×1 dipole]
            Substrate: [1×1 dielectric]
    GroundPlaneLength: Inf
     GroundPlaneWidth: 0.2000
              Spacing: 0.0750
        EnableProbeFeed: 0
                 Tilt: 0
             TiltAxis: [1 0 0]
                 Load: [1×1 lumpedElement]
```

View Antenna Geometry

View the physical construction of infinite ground plane reflector antenna.

```
figure;show(r);
```

dipole over infinite ground plane
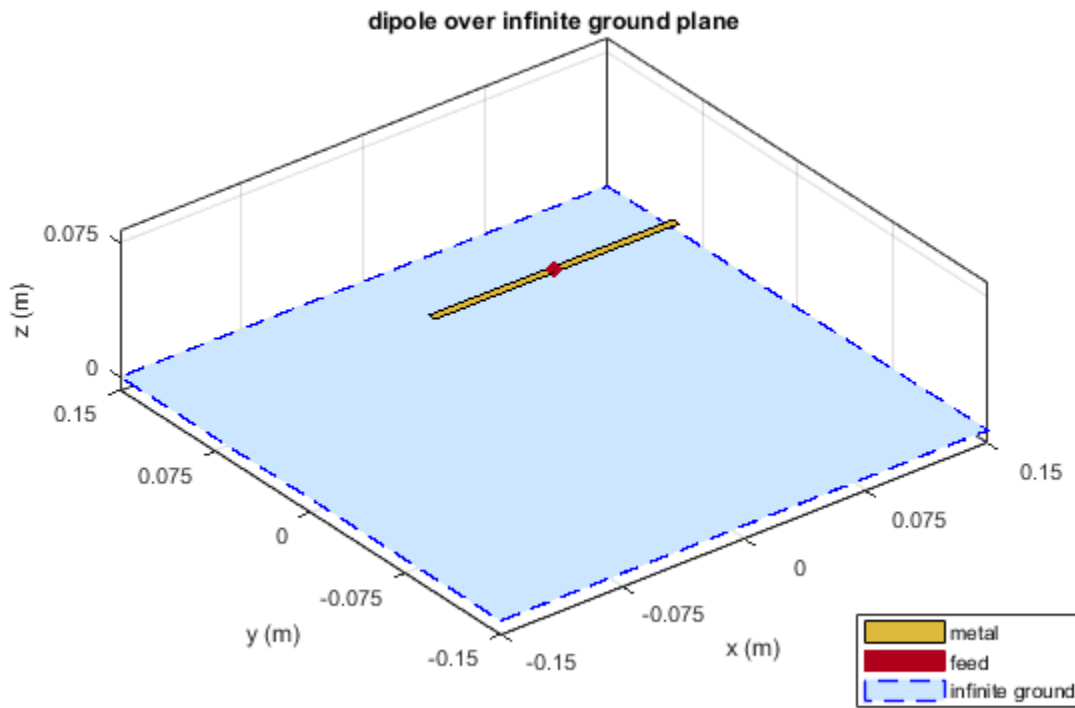
Calculate Impedance of Antenna

Calculate the impedance of reflector antenna over a freqeuncy range of 800MHz to 1GHz.

```
figure;impedance(r,800e6:5e6:1e9);
```
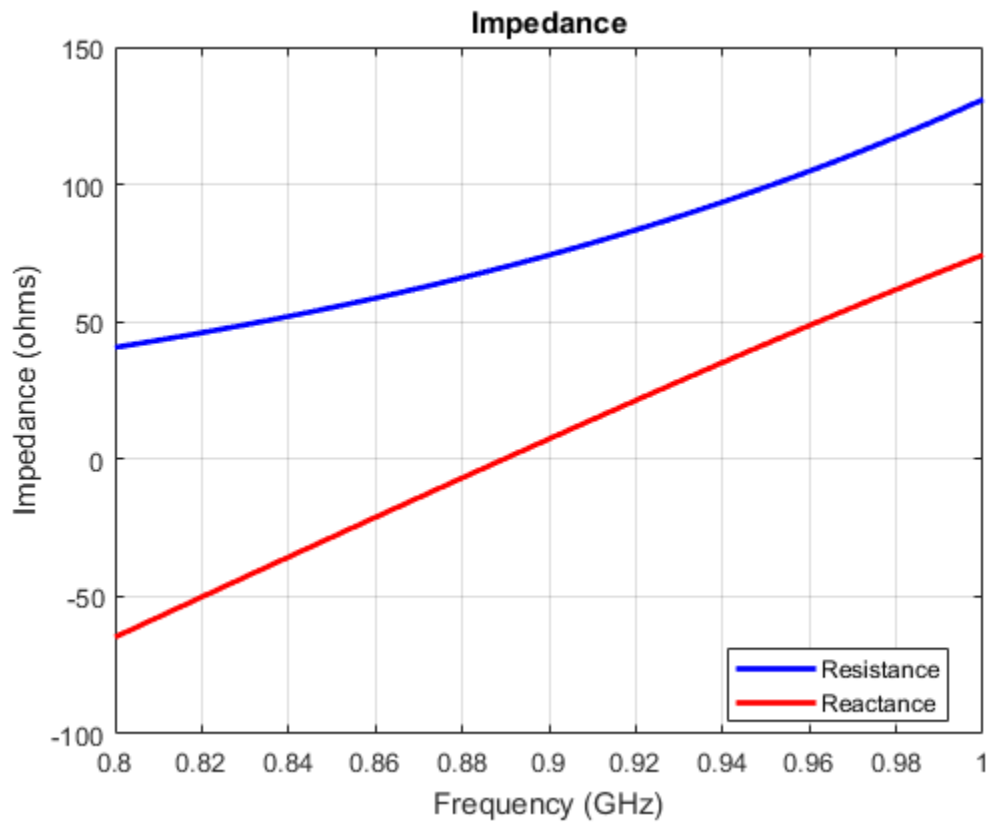
Plot Radiation Pattern of Antenna

Plot the radiation pattern of reflector antenna at a frequency of 900MHz.

```
figure;pattern(r,900e6);
```
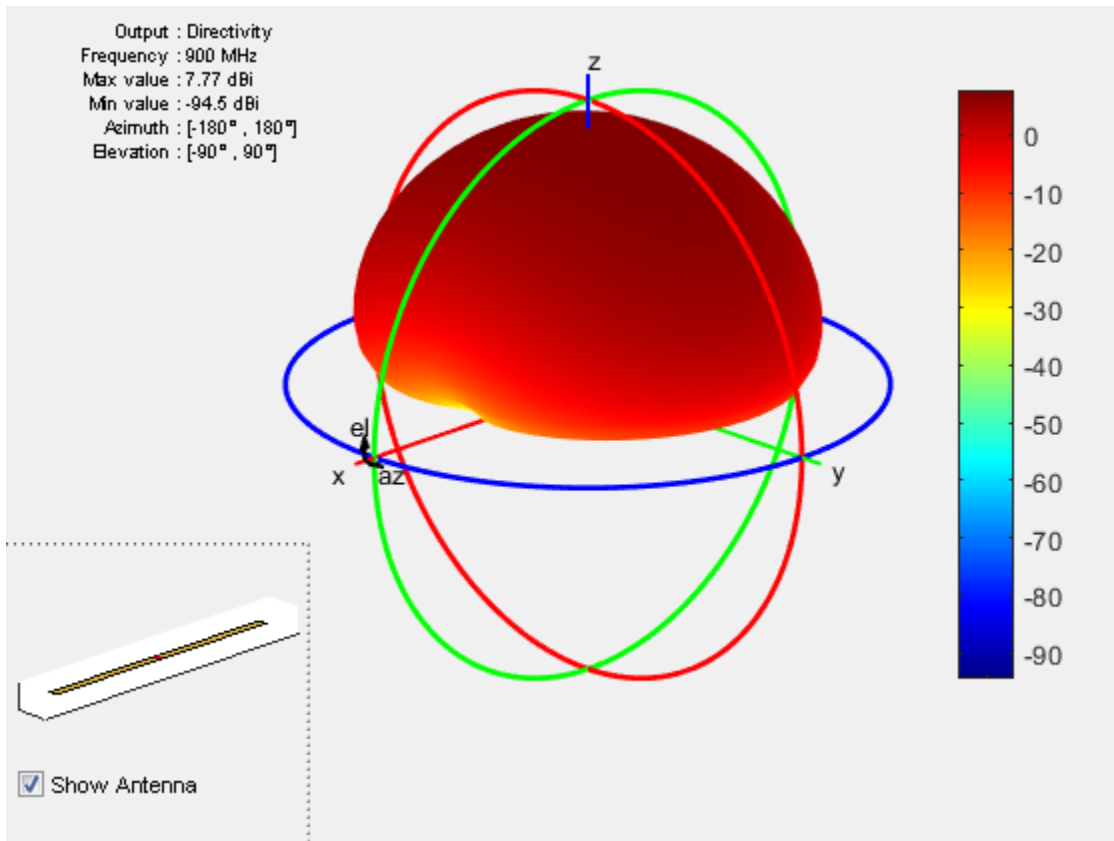
Visualize Antenna Mesh

Mesh and view the infinite ground plane reflector antenna.

```
figure; mesh(r);
```
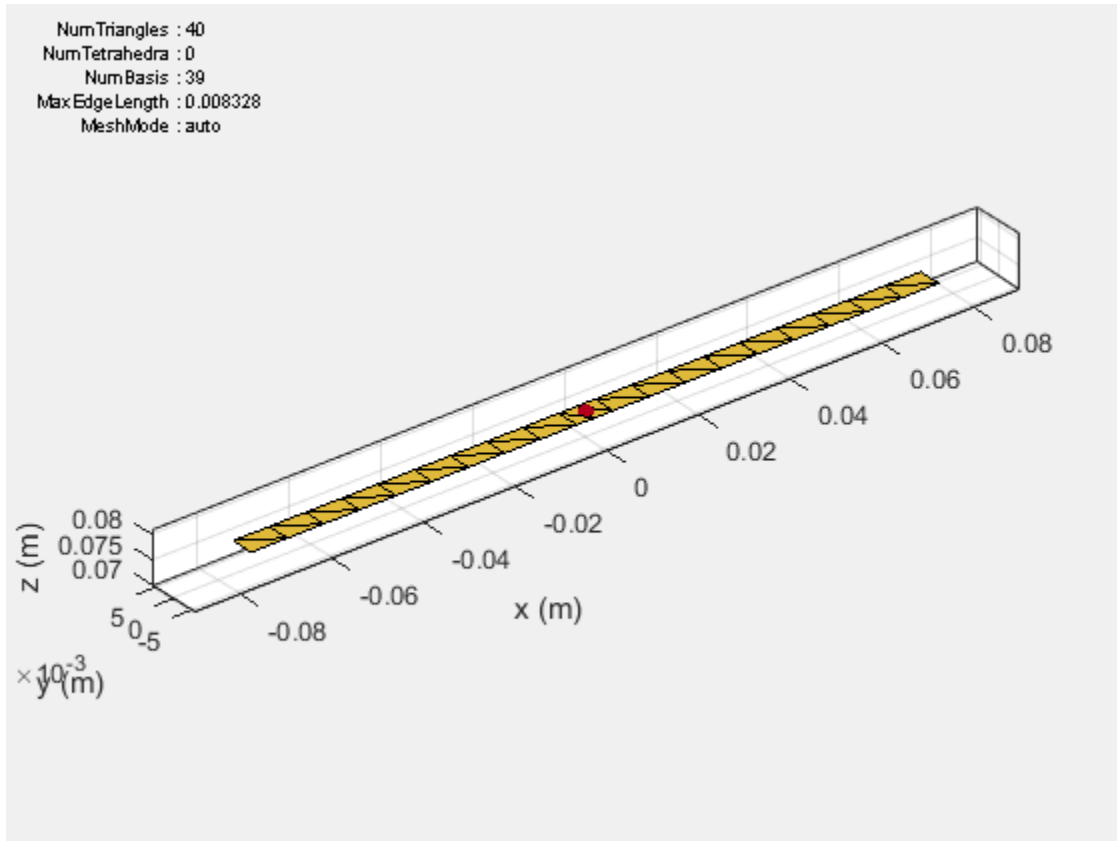
Model Infinite Ground Plane for Unbalanced Antennas

This example shows how to model an infinite ground plane and calculate fundamental antenna parameters for unbalanced antennas.

Create Antenna On Infinite Ground Plane

Create a patch microstrip antenna on a ground plane of infinite length.

```
p = patchMicrostrip('GroundPlaneLength', inf)


p =

  patchMicrostrip with properties:

               Length: 0.0750
                Width: 0.0375
               Height: 0.0060
            Substrate: [1×1 dielectric]
     GroundPlaneLength: Inf
      GroundPlaneWidth: 0.0750
     PatchCenterOffset: [0 0]
            FeedOffset: [-0.0187 0]
                 Tilt: 0
             TiltAxis: [1 0 0]
                 Load: [1×1 lumpedElement]
```

View Antenna Geometry

View the physical construction of the patch microstrip antenna using infinite ground plane.

```
figure;
show(p);
```

patchMicrostrip over infinite ground plane
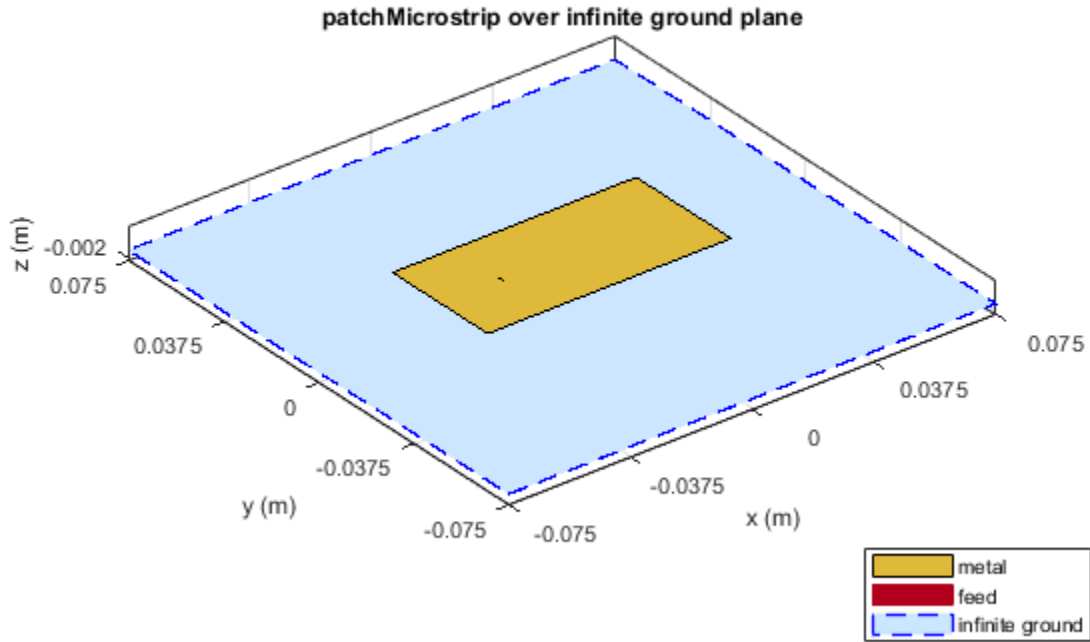
Calculate Impedance of Antenna

Calculate the impedance of the antenna over a freqeuncy range of 1.60 GHz to 1.70 GHz.

```
figure;
impedance(p,1.60e9:5e6:1.70e9);
```
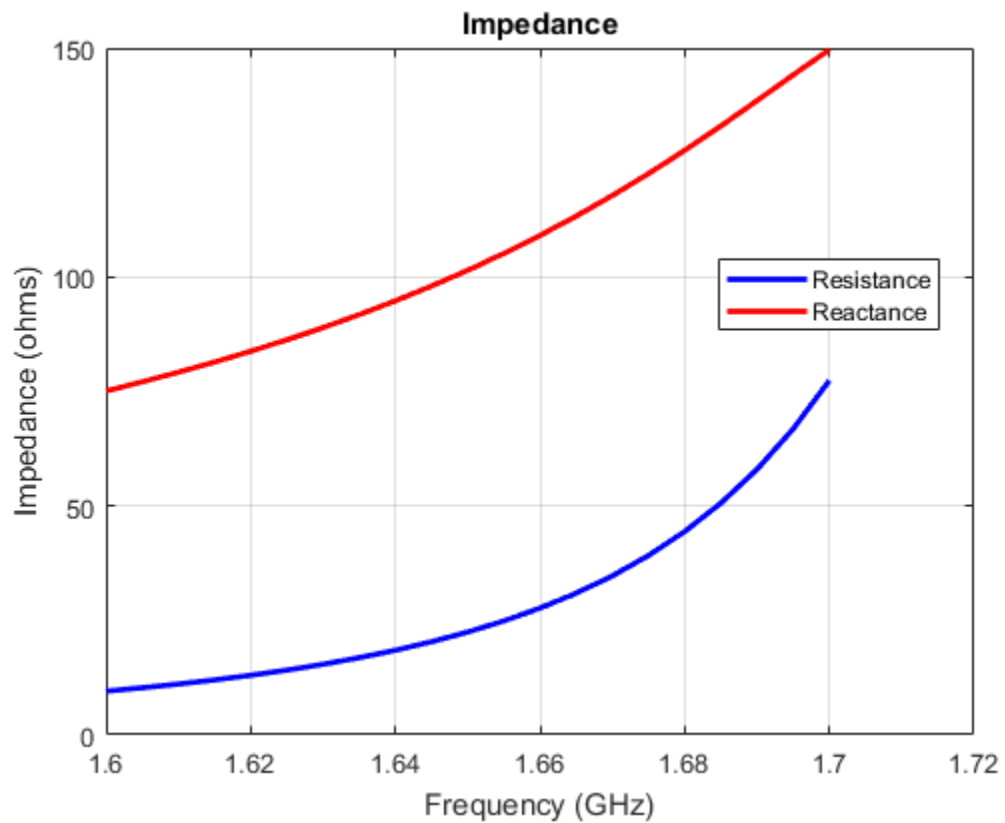
Plot Radiation Pattern of Antenna

Plot the radiation pattern of the antenna at a frequency of 1.67 GHz.

```
figure;
pattern(p,1.67e9);
```
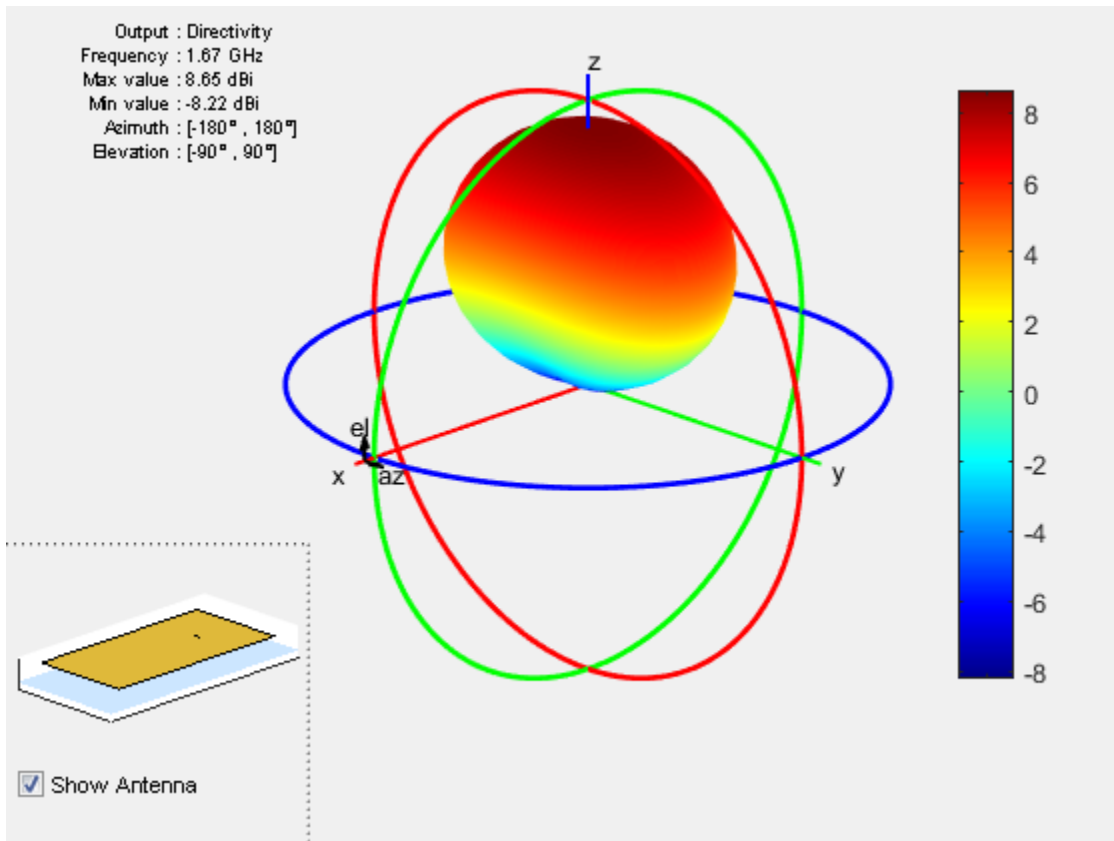
Visualize Antenna Mesh

Mesh and view the infinite ground plane patch microstrip antenna.

```
figure;
mesh(p);
```

2
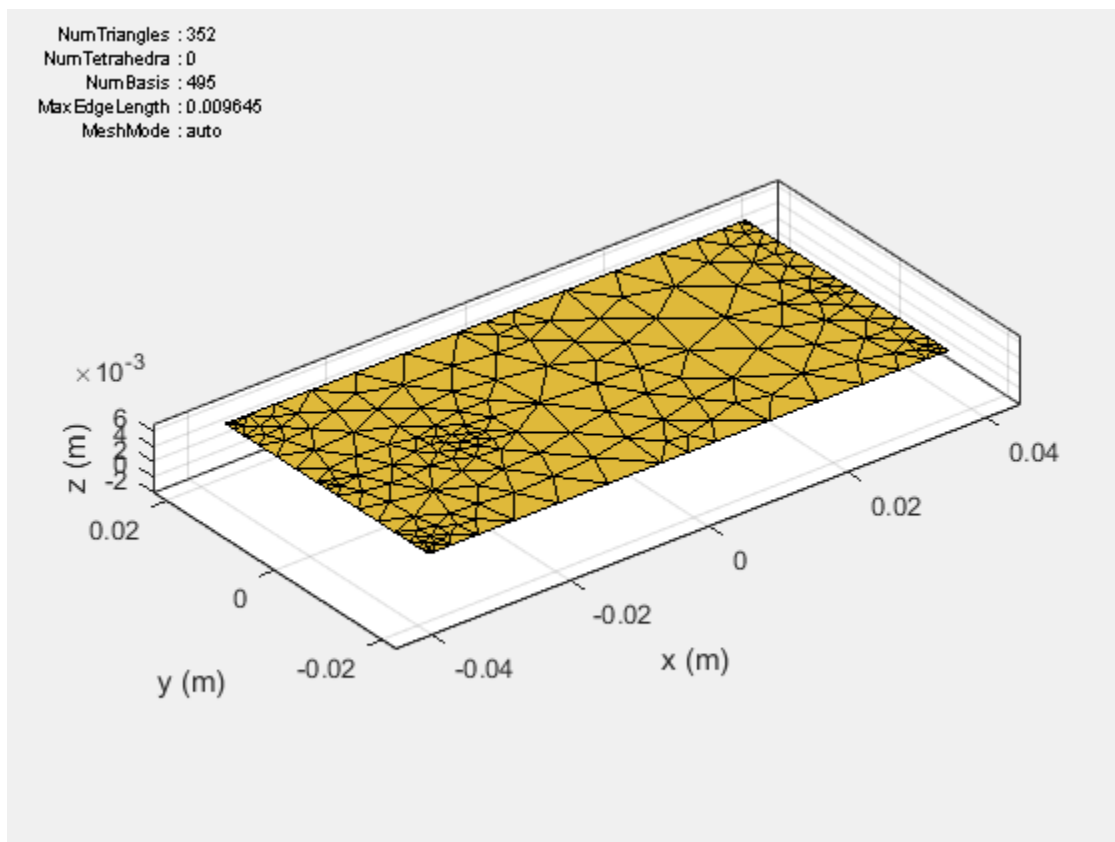
Array Concepts

Mutual Coupling

In this section...

Mutual coupling is the electromagnetic interaction between the antenna elements in an array. The current developed in each antenna element of an array depends on their own excitation and also on the contributions from adjacent antenna elements. Mutual coupling is inversely proportional to the spacing between the different antenna elements in an array. Mutual coupling in an array causes:

- Changes in the radiation pattern of the array
- Changes in the input impedance of the individual antenna elements in an array

To characterize mutual coupling, you can use mutual impedance, S-parameters, a coupling matrix, or an embedded element pattern.

Active or Scan Impedance

Active impedance, or *scan impedance*, is the input impedance of each antenna element in an array, when all elements are excited.
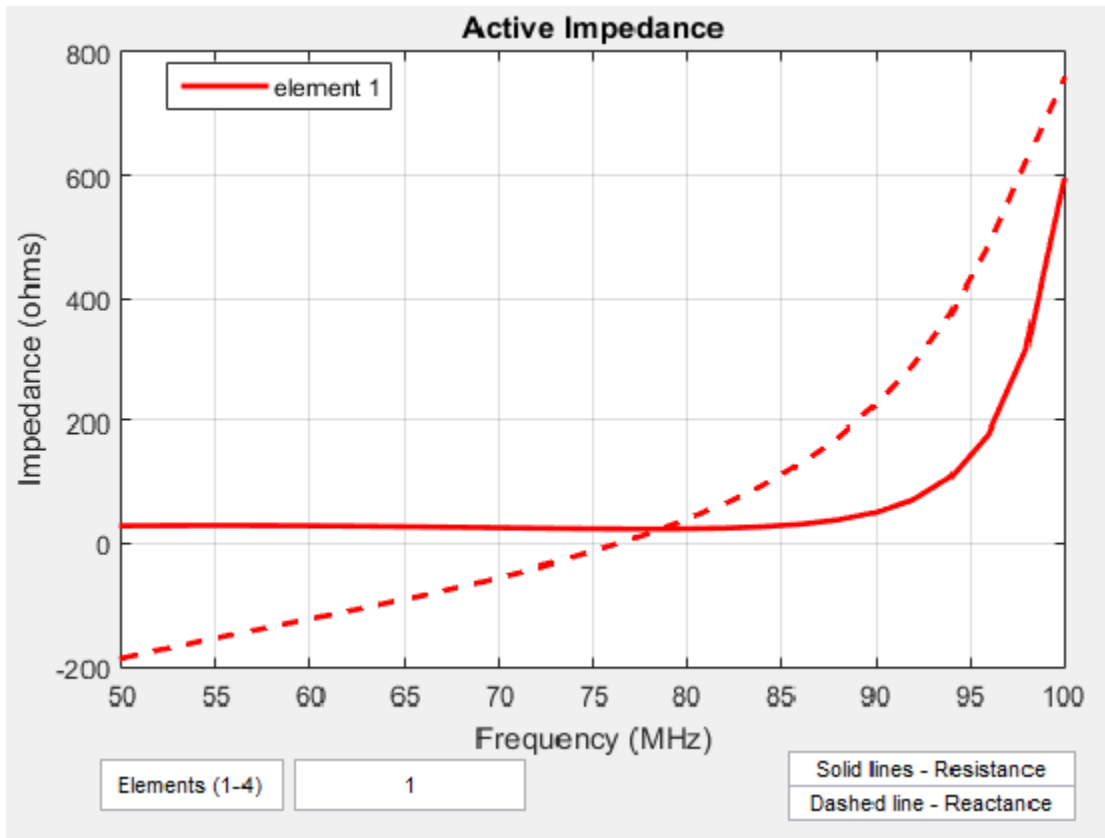
The active impedance of an array depends on:

- Array configuration
- Spacing between elements
- Phase shift applied at each element

Mutual Impedance

The effect of mutual coupling is observed or modeled by varying the space between the antenna elements in the array. Any change in the inter-element spacings changes the *mutual impedance* between the antenna elements. For example, the plot shows the mutual impedance of a two-element dipole array as a function of inter-element spacing.

Mutual Impedance Variation With Spacing
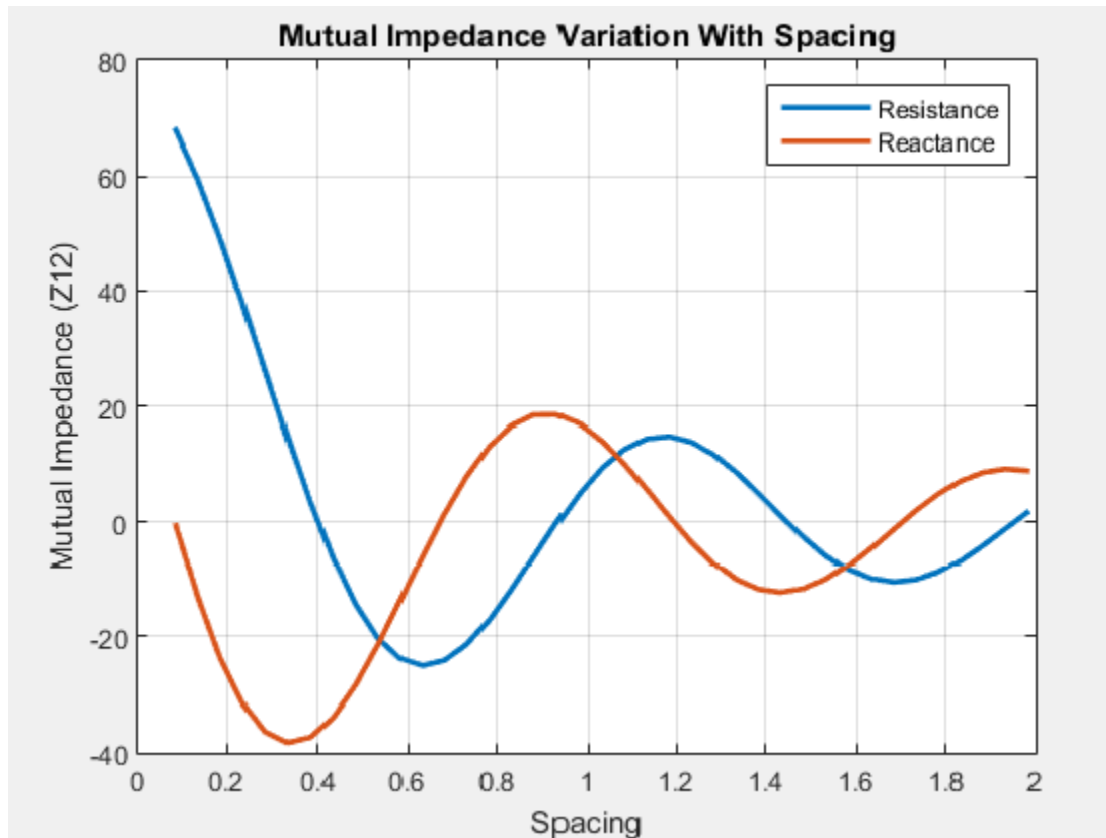
The plot shows that as the spacing between elements increases, the mutual impedance, and hence mutual coupling decreases.

Coupling Matrix

A *coupling matrix* is used to characterize the mutual coupling between the antenna elements at the port level. This matrix is calculated using S-parameters or Z-parameters and is used to decouple the array.

S-Parameter Matrix

To calculate the coupling matrix, you can use the S-parameter matrix. You calculate each column of the S-parameter matrix by feeding the antenna in that column by 1V. Consider

an element array arranged in a 2x2 grid. Visualize the grid and the element numbers using the `layout`.
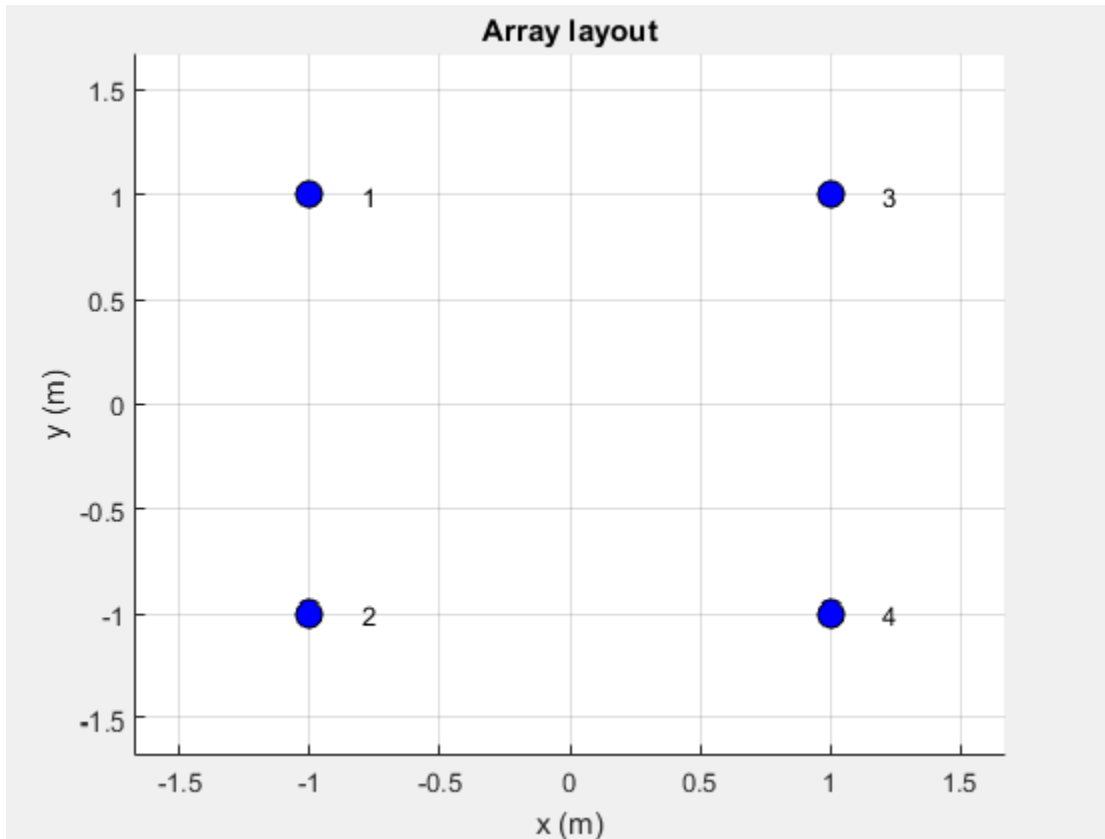
There are four ports in this array. The corresponding S-parameter matrix would be of size 4 x 4:

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{pmatrix}$$

During this measurement, terminate rest of the antenna elements using reference resistance of 50 ohms. The termination resistance is internally set to a default of 50 ohms and can be omitted during the calculation. If the termination is different, specify the resistance value when using this function. The diagonal terms in the matrix shown, represent the self-interaction which is also commonly referred to as the reflection coefficient. The off-diagonal terms capture the mutual coupling between the ports of the antenna.

Use `sparameters` to calculate the S-parameter coupling matrix of an array in Antenna Toolbox.

Pattern Multiplication

Pattern multiplication calculates the complete array pattern as a product of a single element pattern and array factor. Pattern multiplication assumes that all elements in an array have the same radiation pattern. The *array factor* is a function of the number of elements, the geometrical arrangement, the relative magnitude, the relative phases, and the spacings between elements in the array. This calculation is used on large arrays of identical elements.

Isolated Element Pattern

Patterns of individual elements in small arrays vary significantly. Therefore you cannot use pattern multiplication for complete array pattern because isolated element pattern assumes that all elements have the same pattern.

You can calculate the complete array pattern of small arrays by plotting the pattern all the elements separately in a small array. To obtain this pattern, each element is individually excited and the rest of the array elements terminated using reference impedance. The plot shows the radiation pattern of individual elements of 4--element array.
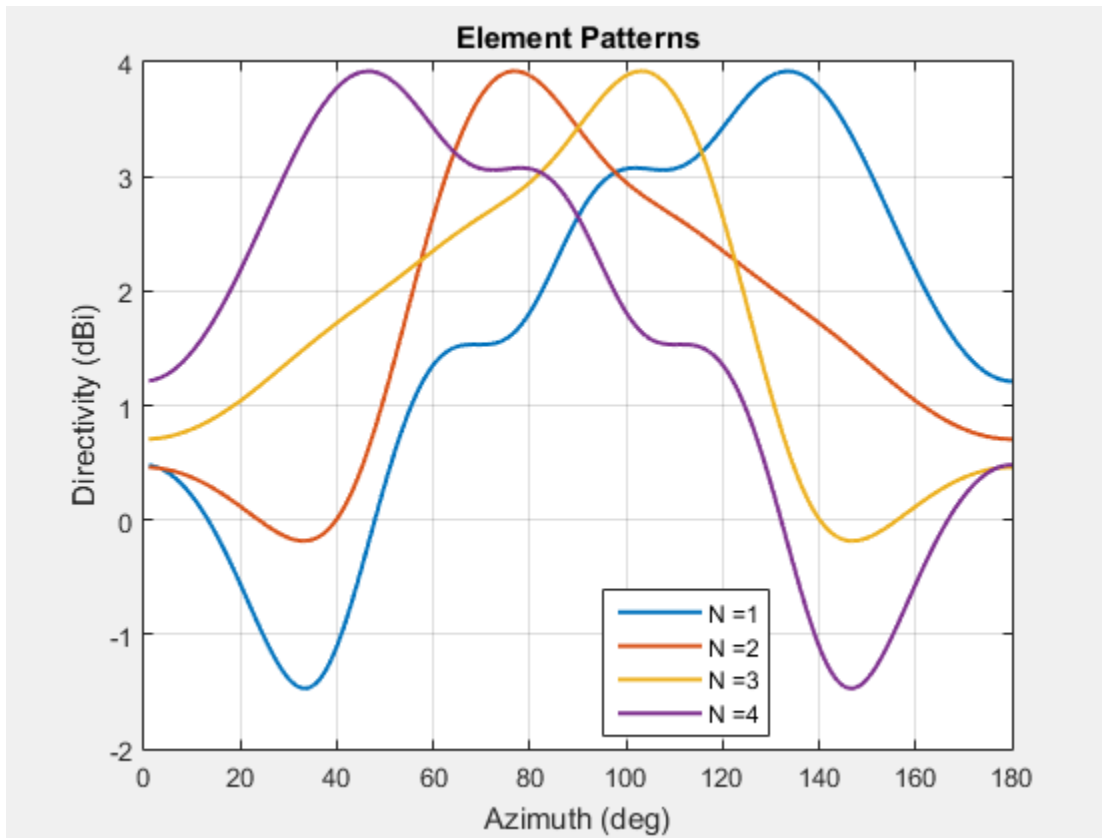
Embedded Element Pattern

The *embedded element pattern* is the pattern of a single element embedded in a finite array, calculated by driving a specific (typically the central) element in the array. The rest of the array elements are terminated using reference impedance. This method is useful for large array because the effect of mutual coupling on the individual element is captured. It is important to note that the edge effects can be ignored since the size of the array is assumed to be very large. It is common to use the center antenna element for this calculation. Due, to the size of the array the radiation patterns of the elements in the array can be approximated with the embedded element pattern instead of the isolated element pattern. Finally, pattern multiplication is used to calculate the complete array pattern.
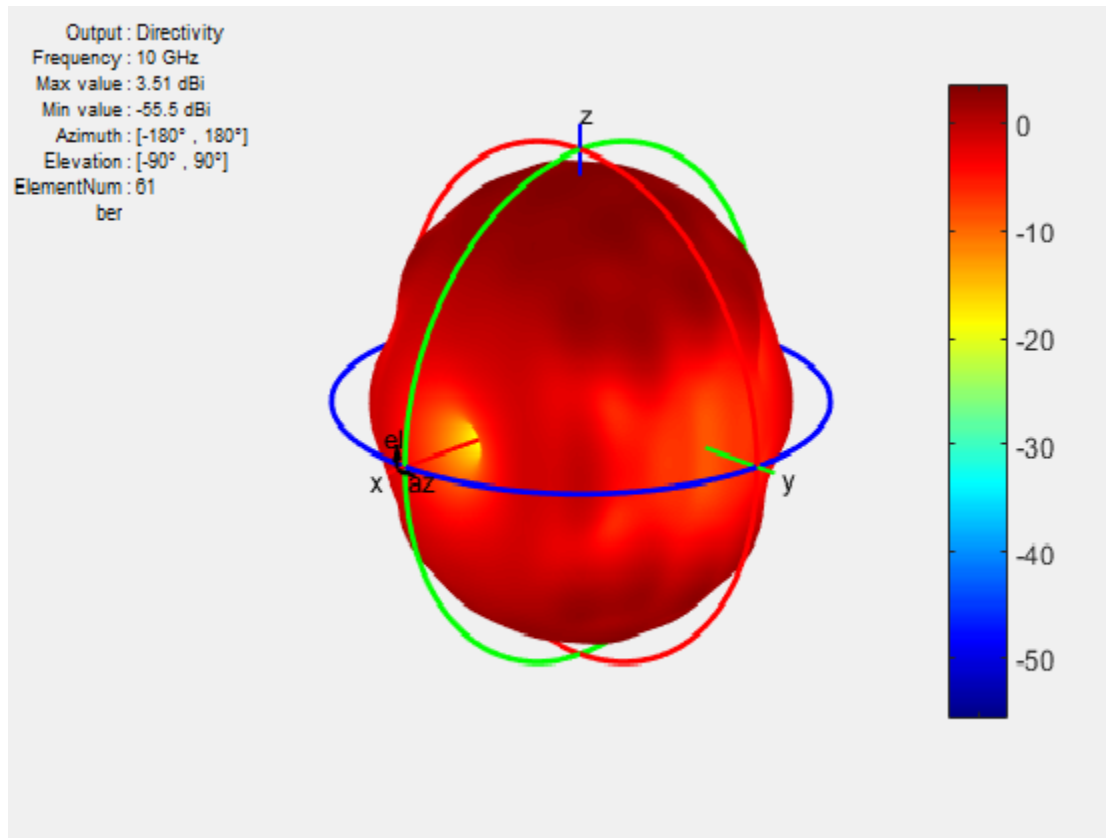
The figure shows the embedded element pattern of a center element in a 11x11 array. You can also calculate the embedded element pattern as a magnitude of electric field.

Isolated element pattern is not recommended for large arrays as this method does not account for the coupling effects of elements around it.

Scan Blindness

In large arrays, it is possible that the array directivity reduces drastically at certain scan angles. At these scan angles, referred to as the blind angles, the array does not radiate the power supplied at its input terminals [3]. The scan blindness can occur while using these common mechanisms:

- Surface wave excitation

- Grating lobe excitation

To detect scan blindness in large finite arrays, study the embedded element pattern. In infinite array analysis, this pattern is known as the array element pattern.

References

[1] Stutzman, W.L. Thiele, G.A. *Antenna Theory and Design*, 3rd Edition. New York: Wiley, 2013, p. 307.

Beamforming

In this section...
"Side Lobe Control" on page 2-10
"Beam Scanning" on page 2-12

Beamforming is the process of generating a directional beam from an antenna array. Achieve beamforming by weighting individual elements by using sidelobe control or beam scanning.

Side Lobe Control

Side lobes are undesired and lead to reception or transmission of energy in unwanted directions. *Side lobe control* in an array is achieved using amplitude taper or amplitude weighting. Amplitude tapering changes the excitation amplitude of each element in the array. Minor lobe levels are controlled using amplitude taper that runs from the center of the array to the end of the array. Smoother amplitude tapering gives larger small side lobe levels but half-power beamwidth. These arrays are non-uniformly excited arrays.

Consider a linear array of 21 elements. Without amplitude tapering, the array contains unwanted side lobes in the desired direction:

Output : Directivity
Frequency : 70 MHz
Max value : 16.9 dBi
Min value : -19.8 dBi
Azimuth : [0° , 180°]
Elevation : 0°

By using amplitude tapering on the linear array, you control the side lobes and achieve a better main beam in the desired direction:
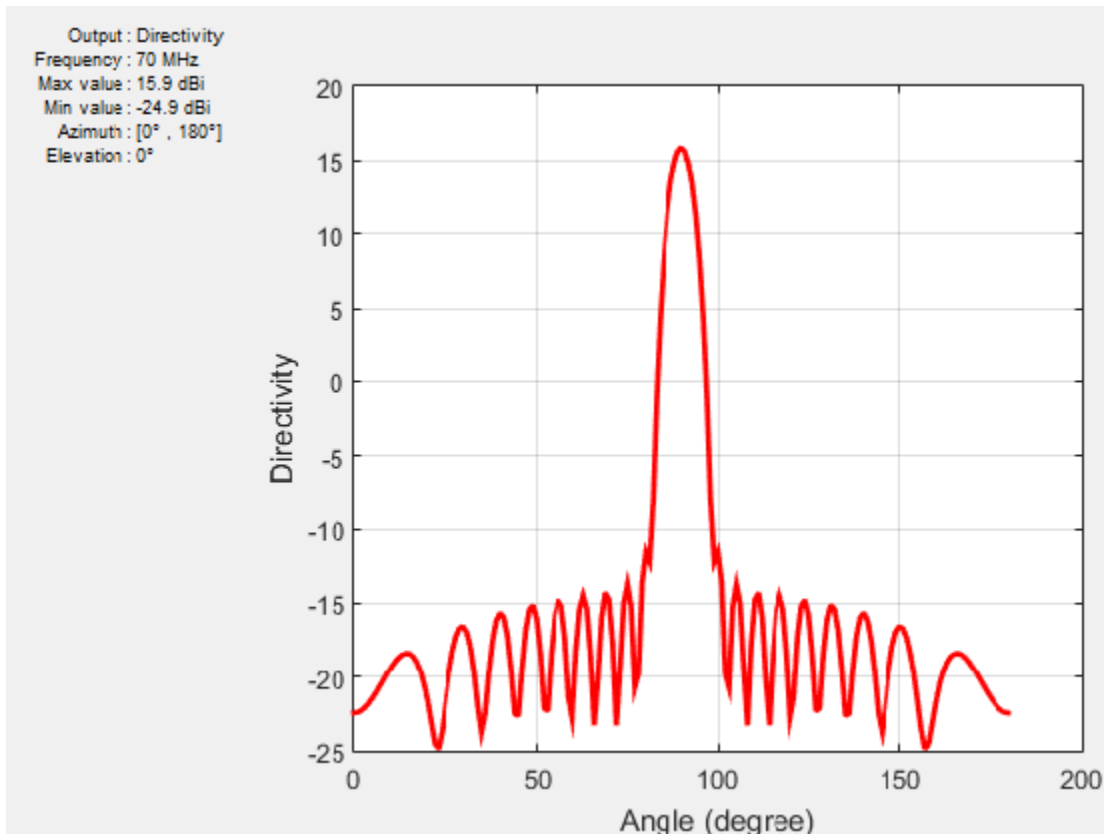
Beam Scanning

Beam scanning is the movement of a radiation pattern in space. You can achieve beam scanning by controlling the progressive phase difference between the elements to direct the beam in any desired direction. The phase shift changes the phase of the excitation currents of each element in an array. You can achieve this phase shift using phase sifter devices, a time delay, frequency scanning, beam switching, or digital beamforming.

Consider a linear array of 21 elements where peak directivity is shown without beam scanning:

Output : Directivity
Frequency : 70 MHz
Max value : 15.9 dBi
Min value : -24.9 dBi
Azimuth : [0° , 180°]
Elevation : 0°
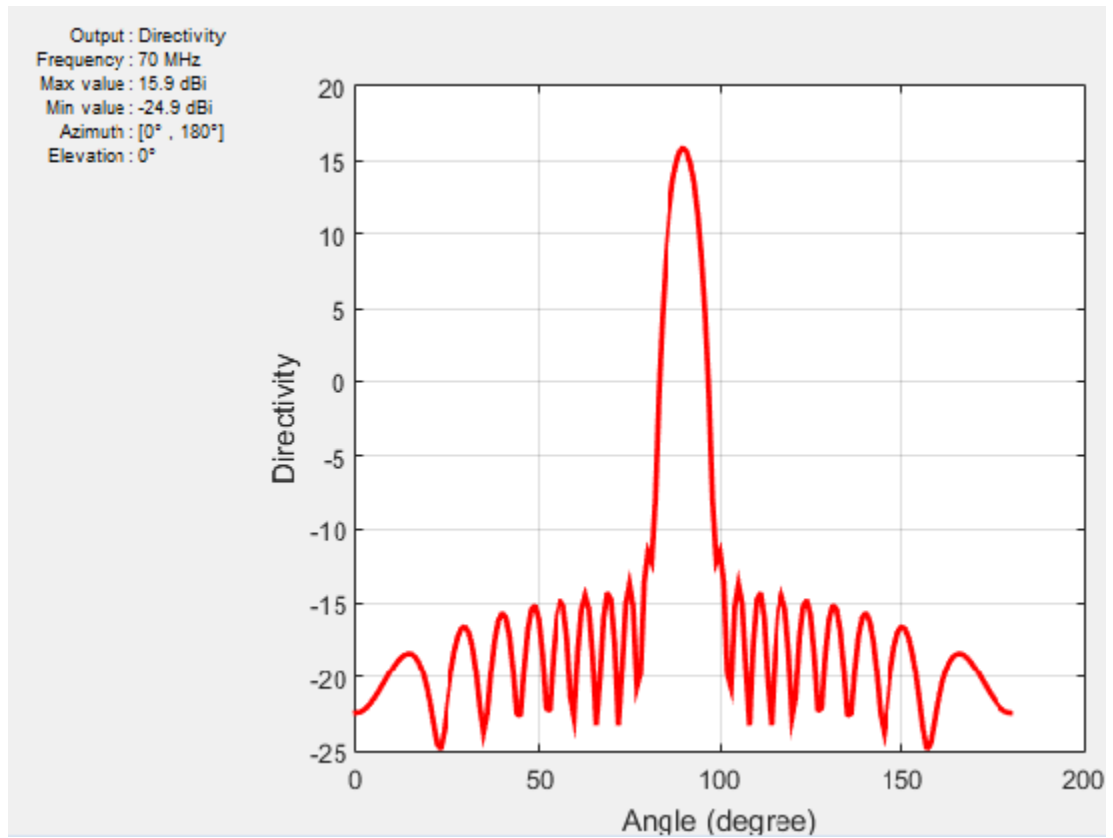
To scan the beam at a specific angle, use phase shift on the linear array:
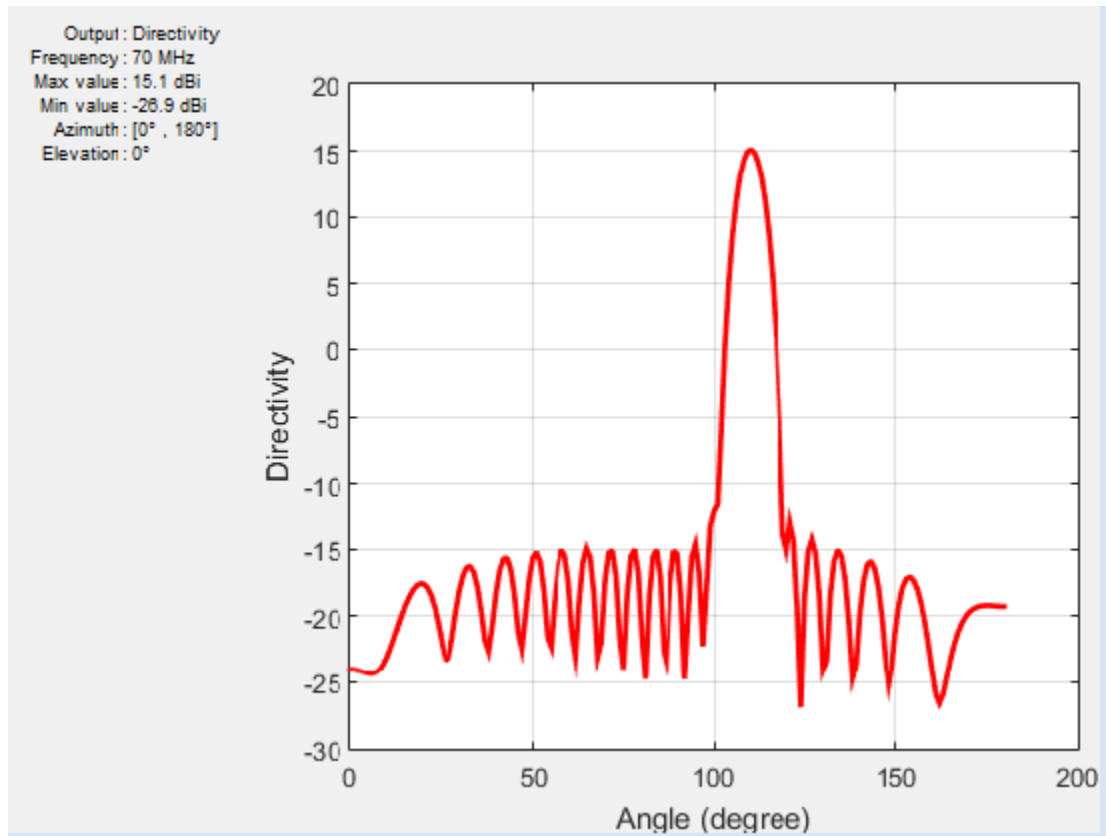
References

[1] Stutzman, W.L. Thiele, G.A. *Antenna Theory and Design*, 3rd Edition. New York: Wiley, 2013, p. 307.

Grating Lobes

Grating lobes are the maxima of the main beam, as predicted by the pattern multiplication theorem. When the array spacing is less than or equal to $\lambda/2$, only the main lobe exists in the visible space, with no other grating lobes. Grating lobes appear when the array spacing is greater than $\lambda/2$. For large spacing, grating lobes can appear in the visible space even at a zero scan angle.

Consider a linear array of seven elements spaced one-half wavelength apart. This array contains no grating lobes.

Output: Directivity
Frequency: 70 MHz
Max value: 11.3 dBi
Min value: -15.2 dBi
Azimuth: [0° , 180°]
Elevation: 0°

Spacing is half-wavelength
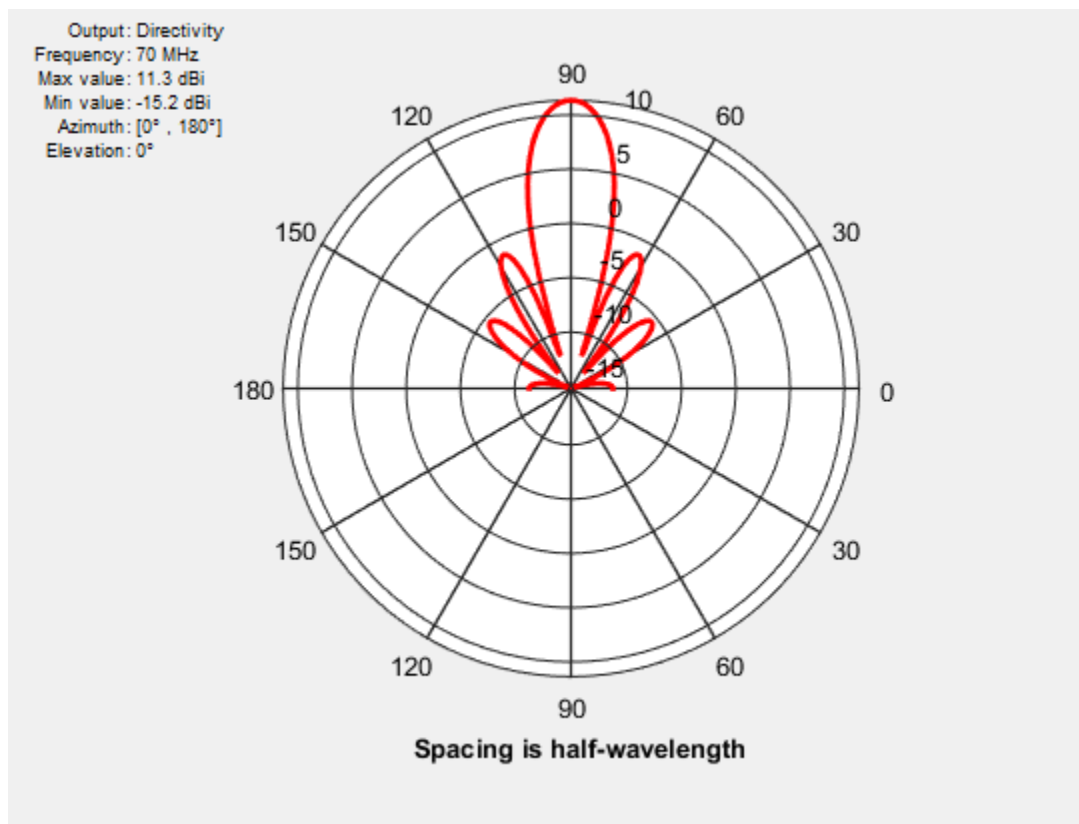
When you increase the element spacing to one wavelength, the number of side lobes increases.

Output: Directivity
Frequency: 70 MHz
Max value: 12.7 dBi
Min value: -24.2 dBi
  Azimuth: [0° , 180°]
Elevation: 0°

Spacing is one-wavelength
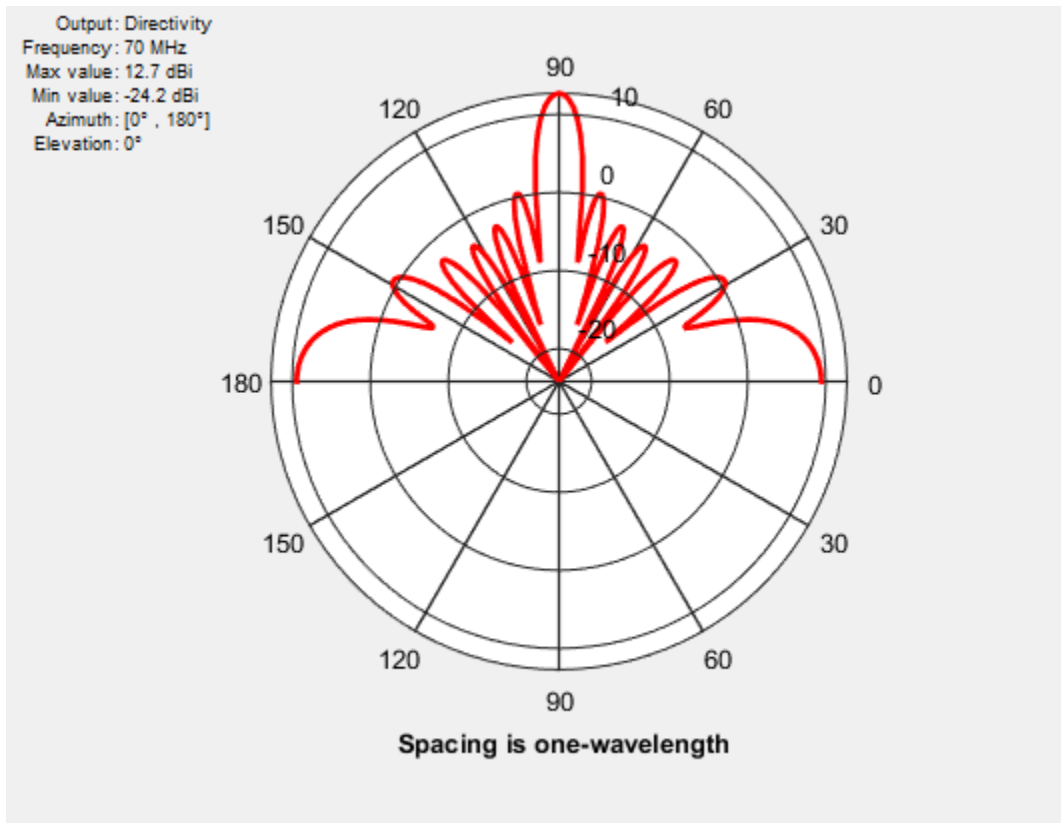
When you increase the element spacing to 1.5 wavelengths apart, the grating lobes appear in the visible region.

Output: Directivity
Frequency: 70 MHz
Max value: 10.5 dBi
Min value: -17.6 dBi
Azimuth: [0° , 180°]
Elevation: 0°

Spacing is one and half-wavelength
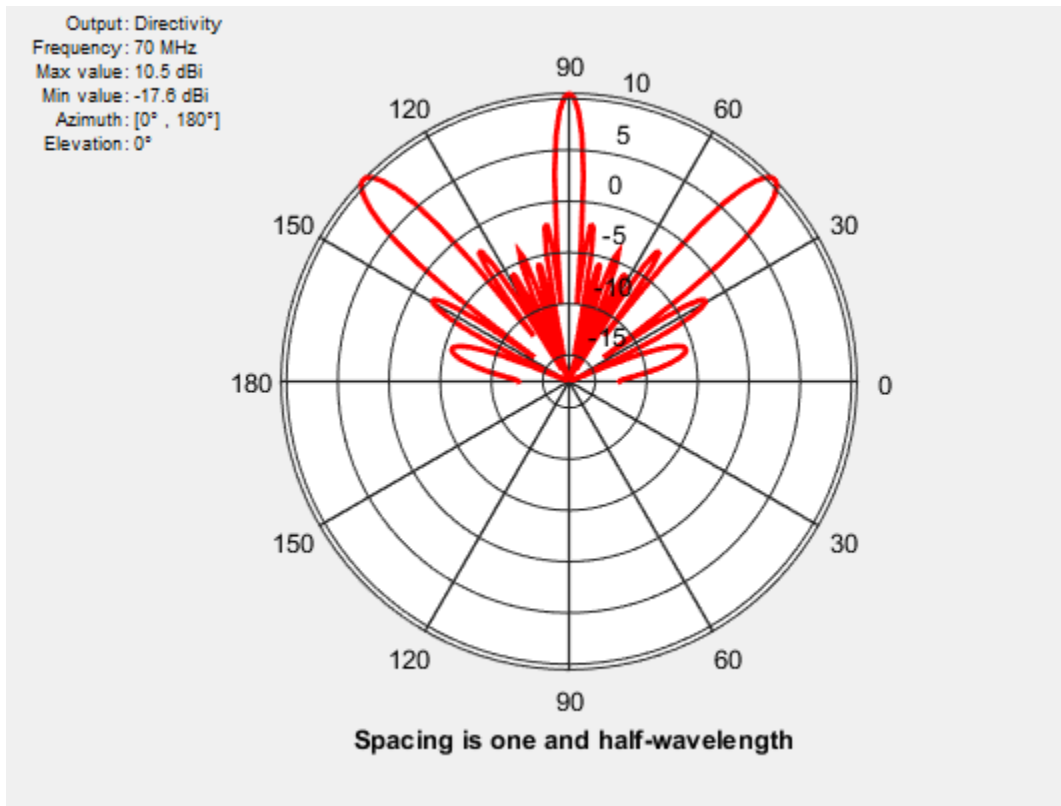
The grating lobes move in or out of the visible region depending on the spacing between the elements and the relative phase between them.

References

[1] Stutzman, W.L. Thiele, G.A. *Antenna Theory and Design*, 3rd Edition. New York: Wiley, 2013, p. 307.

Correlation Coefficient
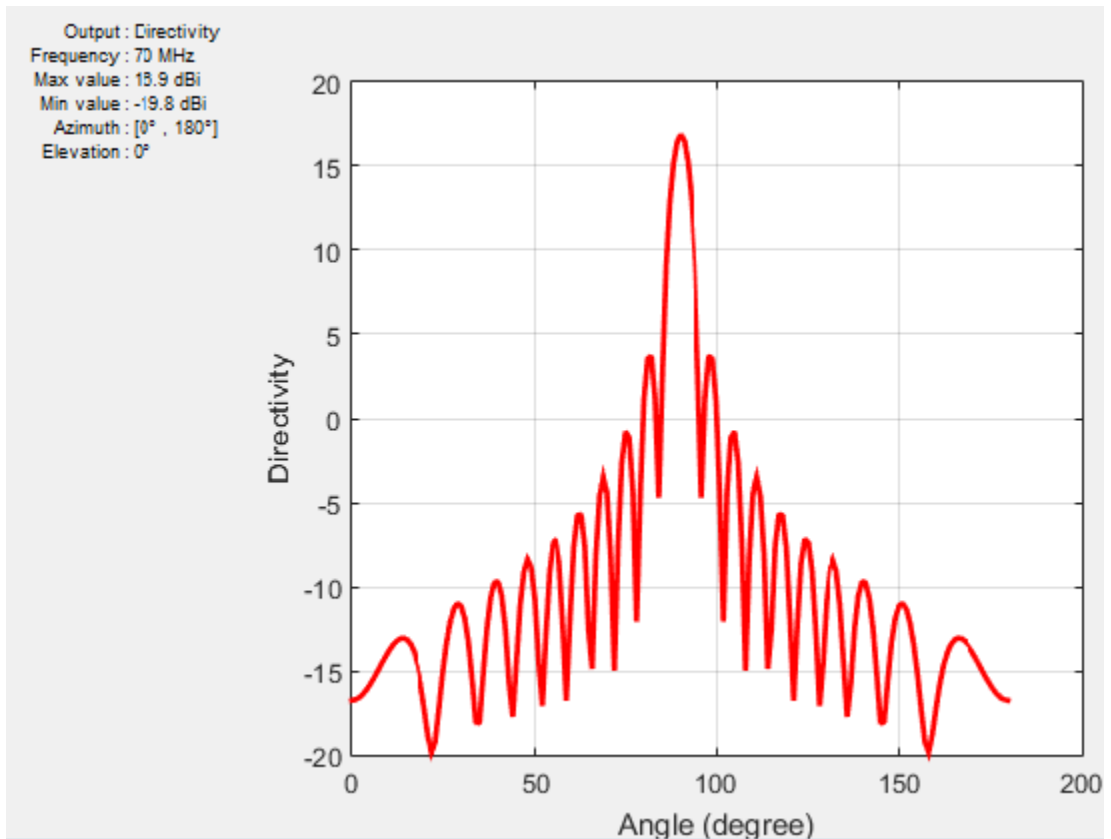
In this section...
"Far-Field Radiation Pattern" on page 2-18
"S-Parameter Characterization" on page 2-18

The *correlation coefficient* is the relationship between the incoming signals at the antenna ports in an array. Mutual coupling in array systems degrades the performance of the array. The correlation coefficient between antennas is used as a performance metric in multiple-input multiple-output (MIMO) systems to quantify the system performance and efficiency of the antennas. By using the correlation coefficient, a MIMO system designer is able to understand the level of coupling that exists between the antenna ports in the system. To minimize the mutual coupling would imply to reduce the correlation coefficient between the pairs of ports. Antenna designers use two approaches to the calculate correlation coefficient: the far-field Radiation pattern and S-parameters.

Far-Field Radiation Pattern

The correlation coefficient of a two antenna array system is:

$$\rho_e = \frac{\left| \int\limits_{4\pi} \int [\overrightarrow{F_1}(\theta,\phi) \bullet \overrightarrow{F_2}(\theta,\phi)] d\Omega \right|^2}{\int\limits_{4\pi} \int \left| \overrightarrow{F_1}(\theta,\phi) \right|^2 d\Omega \int\limits_{4\pi} \int \left| \overrightarrow{F_2}(\theta,\phi) \right|^2 d\Omega}$$

where $\overrightarrow{F_i}(\theta,\phi)$ is the radiation pattern of the antenna system when port i is excited. Computing the correlation coefficient using this formula, requires the radiation pattern of the antenna. This approach is hard and time consuming.

S-Parameter Characterization

Antenna Toolbox™ uses the S-parameter characterization to calculate correlation between antenna elements in an array. This approach is simpler than the far-field approach because the S-parameter calculation does not use the radiation patterns of the antennas. Correlation coefficient is calculated using S-parameter by using:

$$\rho_e = \frac{\left|S_{11}^* S_{12} + S_{21}^* S_{22}\right|^2}{(1-(\left|S_{11}\right|^2 + \left|S_{21}\right|^2))(1-(\left|S_{22}\right|^2 + \left|S_{12}\right|^2))}$$

The advantages of this method are quick analysis and broadband correlation results. However, this approach assumes that the antennas are lossless and that incoming waves are uniformly distributed. To calculate and plot the correlation between antennas in an array, use the `correlation` function in Antenna Toolbox™.

References

[1] Blanch, S. Romeu, J. and Corbella, I. *Exact Representation of antenna system diversity performance from input parameter description*

[2] Stutzman, W.L. Thiele, G.A. *Antenna Theory and Design*, 3rd Edition. New York: Wiley, 2013, p. 307.

Infinite Arrays

What Are Infinite Arrays?

Infinite arrays are rectangular arrays of infinite extent. In an infinite array, a single element called a *unit cell*, is repeated uniformly an infinite number of times along a plane.

Infinite Array Analysis

All arrays used in real-world scenarios are finite. But antenna arrays used in radio astronomy, air defense, or surveillance radar can have more than 1000 antenna elements. In such large arrays, the electromagnetic analysis of each element is tedious and time consuming.

Infinite array analysis ignores the effect of truncation (edge effect) at array edges. The method analyzes the behavior of the active antenna element as a function of frequency and scan. The goal of infinite array analysis is to extract the behavior of the active antenna element embedded in the array.

Assumptions

For infinite array analysis, array size must be greater than 10x10. The technique makes other assumptions:

- Each element is identical.

- Each element is uniformly excited in amplitude.
- All elements are spaced uniformly in two dimensions.

Infinite Array Solver

To model an infinite array, the method of moments (MoM) formulation is changed to account for the infinite behavior by replacing Green's functions with periodic Green's functions. The periodic Green's function is an infinite double summation.

Green's Function	Periodic Green's Function
$$g = \frac{e^{-jkR}}{R}$$ $$R = \lvert \vec{r} - \vec{r}' \rvert$$	$$g_{\text{periodic}} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{j\phi_{mn}} \frac{e^{-jkR_{mn}}}{R_{mn}}$$ $$R_{mn} = \sqrt{(x - x' - x_m)^2 + (y - y' - y_n)^2 + (z - z')^2}$$ $$\phi_{mn} = k(x_m \sin\theta\cos\varphi + y_n \sin\theta\cos\varphi)$$ $$x_m = m \cdot d_x, \quad y_n = n \cdot d_y$$

d_x and d_y are the ground plane dimensions that define the x and y dimensions of the unit cell. θ and $\varPhi$ are the scan angles.

The periodic Green's function has an additional exponential term added to the infinite sum. The $\varPhi_{mn}$ term accounts for the scanning of the infinite array. The periodic Green's function also accounts for the effect of mutual coupling.

Create Infinite Array Using Antenna Toolbox

To create an infinite array, use the infiniteArray object to repeat a single antenna element (unit cell), infinitely along the X-Y plane. The `layout` function displays a typical unit cell.

```
infarray = infiniteArray;
layout(infarray)
```

Infinite Array layout
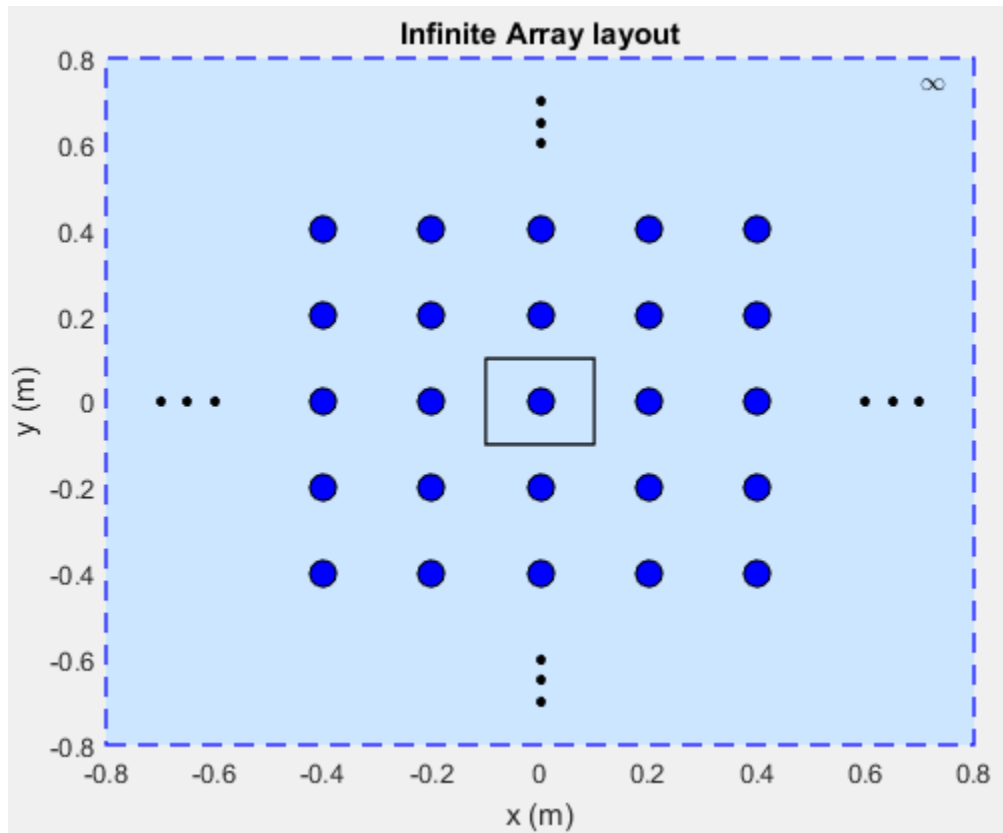
Choose a Unit Cell

You can use any antenna from the Antenna Toolbox as the unit cell. The unit cell requires a ground plane to specify the boundaries. You can use a reflector to back antennas that do not have a ground plane.

The default reflector properties are:

```
r = reflector

 reflector with properties:

            Exciter: [1x1 dipole]
    GroundPlaneLength: 0.2000
```

```
   GroundPlaneWidth: 0.2000
            Spacing: 0.0750
               Tilt: 0
           TiltAxis: [1 0 0]
```

The default unit cell in an infinite array is a reflector that has a dipole as an exciter. The Spacing property gives the distance between the reflector and the exciter. The default infinite array properties are:

```
infarray = infiniteArray

 infiniteArray with properties:

          Element: [1x1 reflector]
      ScanAzimuth: 0
    ScanElevation: 90

show (infarray)
```


Unit cell of dipole over a reflector in an infinite Array
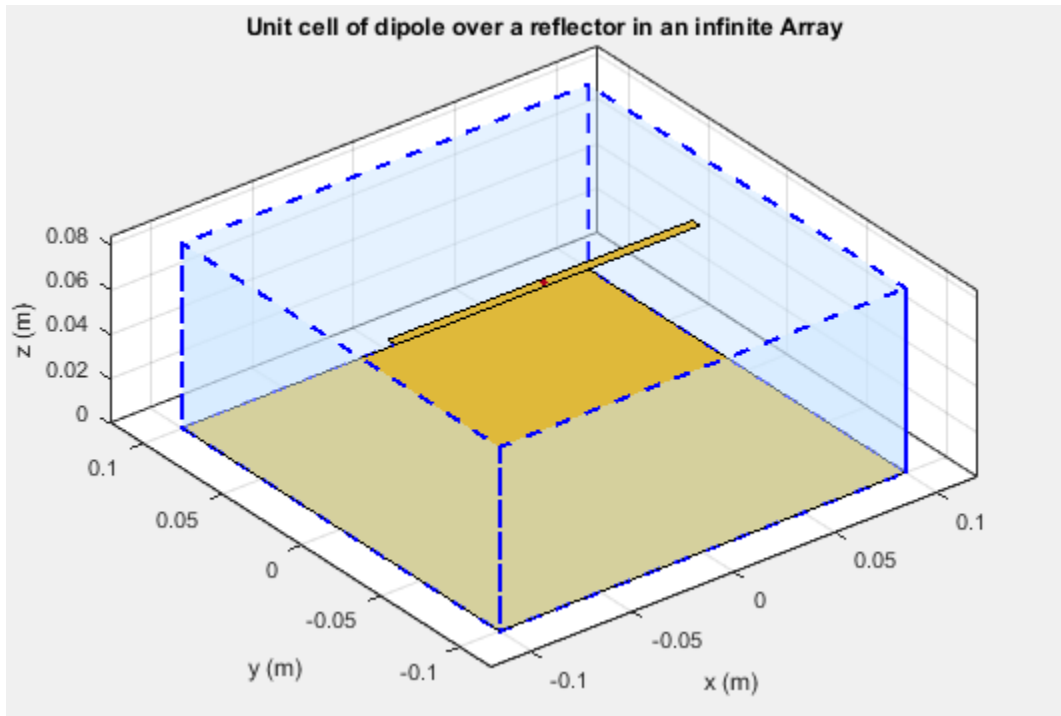
The dotted blue box bounds the unit cell. Ground plane length and ground plane width of the unit cell are the dimensions of the antenna element of the infinite array.

An antenna with a ground plane, such as a microstrip patch antenna, is specified directly as an `Element` of an infinite array.

```
infarray = infiniteArray('Element', patchMicrostrip)

infarray =
  infiniteArray with properties:

           Element: [1x1 patchMicrostrip]
        ScanAzimuth: 0
     ScanElevation: 90
```

```
show(infarray)
```


Unit cell of patchMicrostrip in an infinite Array
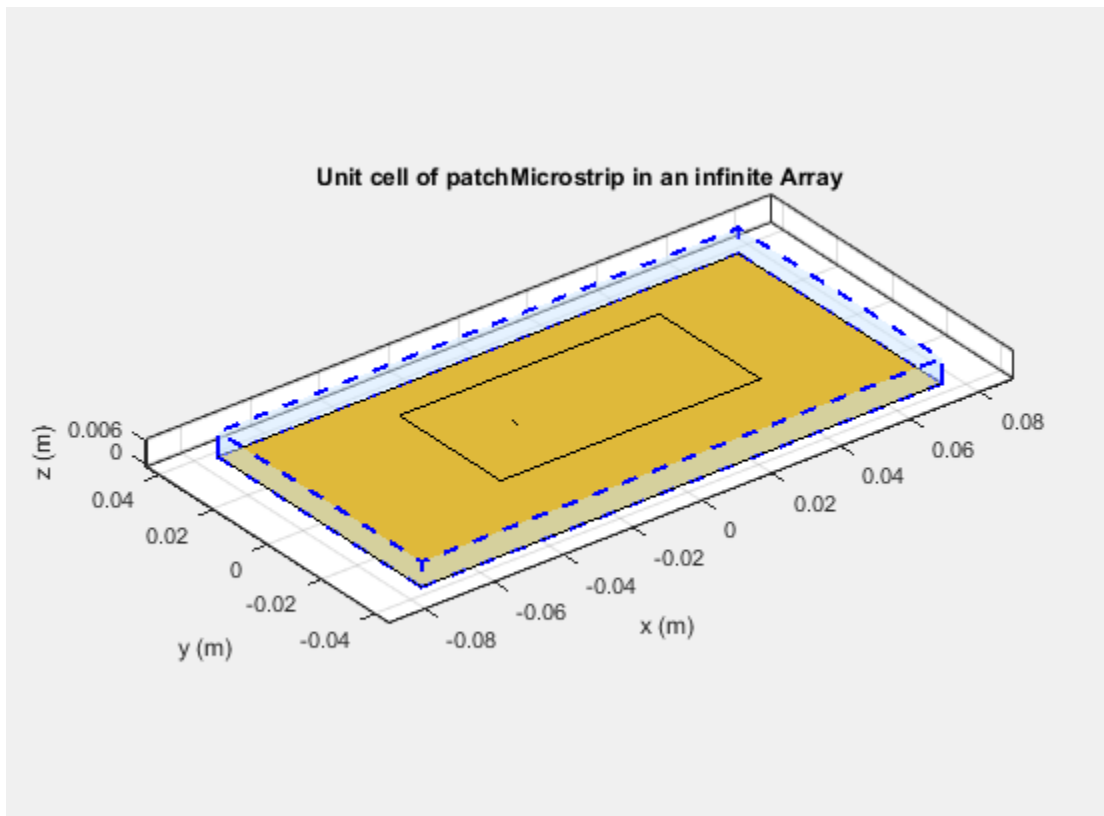
The Antenna Toolbox infinite array is located in the X-Y plane. Unit cells consisting of antennas with ground planes are also located in the X-Y plane. For antennas used as unit cells, such as the one in this example, you ignore the value of the `Tilt` property.

Scan Infinite Arrays

You scan a finite array by specifying the appropriate phase shift for each antenna element. In Antenna Toolbox, you specify the scan angle (in azimuth and elevation) and frequency for infinite array analysis. By default, an array always scans at boresight (azimuth = 0 degrees and elevation = 90 degrees).

```
infarray = infiniteArray

 infiniteArray with properties:

          Element: [1x1 reflector]
       ScanAzimuth: 0
     ScanElevation: 90
```

To change the scan angles, change the values of `ScanAzimuth` and `ScanElevation`.

Scan Impedance and Scan Element Pattern

To calculate the scan impedance for an infinite array, use the `impedance` function as a function of scan angle. Fixing the scan angle pair and sweeping the frequency variable reveals the frequency dependency in the scan impedance. Because `ScanAzimuth` and `ScanElevation` are scalar values, you must a `for`-loop to calculate the complete scan impedance of the array. For more information on calculating the scan impedance and the scan element pattern see, "Infinite Array Analysis".

Scan Element Pattern

To calculate the scan element pattern using scan impedance, use these expressions:

$$g_s(\theta) = \frac{4 R_g R_{\text{iso}} g_{\text{iso}}(\theta)}{\left| Z_s(\theta) + Z_g \right|^2}$$

- R_g — Resistance of generator
- Z_g — Impedance of generator

- Z_s — Scan impedance
- $g_{iso}(\theta)$ — Pattern of isolated element
- R_{iso} — Resistance of isolated element

The scan element pattern can also be expressed in terms of the reflection coefficient, $\Gamma(\theta)$:

$$g_s(\theta) = \frac{4 R_{iso} g_{iso}(\theta)}{R_s(\theta)} \left(1 - \left| \Gamma(\theta) \right|^2 \right)$$

The Antenna Toolbox software calculates the scan element pattern of a finite array by driving just a single element. You terminate all the other elements using a suitable impedance. The resulting element pattern includes mutual coupling and is valid for all scan angles.

Compare Scan Element Pattern of Finite and Infinite Arrays

Case 1: Compare finite array and infinite array with unit cell of dimensions $0.5\lambda \times 0.5\lambda$

To calculate the scan element pattern of the finite arrays, first, create a reflector-backed dipole. Set the dipole dimensions to $\text{Length}(L) = 0.495\lambda$ and $\text{Width}(W) = \lambda / 160$ and the ground plane dimensions to $0.5\lambda \times 0.5\lambda$. Place the dipole at a distance of $h = \lambda/4$ from the reflector. The ground plane dimensions set the boundaries of the unit cell. Create finite arrays of sizes 11x11, 15x15, and 17x17 using this unit cell.

For finite arrays, calculate the scan element pattern by driving a single element in the array. Terminate all other finite array elements using the broadside resistance of the infinite array. For an infinite array with the unit cell of dimensions $0.5\lambda \times 0.5\lambda$, the broadside resistance is 176 Ω. Calculate the scan element pattern for E-, D-, and H-planes of all three finite arrays.

To calculate the scan element pattern of an infinite array, create an infinite array using the same unit cell and the infiniteArray class. Calculate the scan impedance for three scan planes: E, D, and H. Compute the pattern of the isolated element (dipole backed by reflector). Finally, use the equations from the previous section to generate the scan element pattern for the infinite array.

Perform all analysis at 10 GHz. To compare the patterns of finite and infinite array, overlay them on the same plot.

E-Plane (az = 0 deg.) Power Pattern

D-Plane (az = 0 deg.) Power Pattern
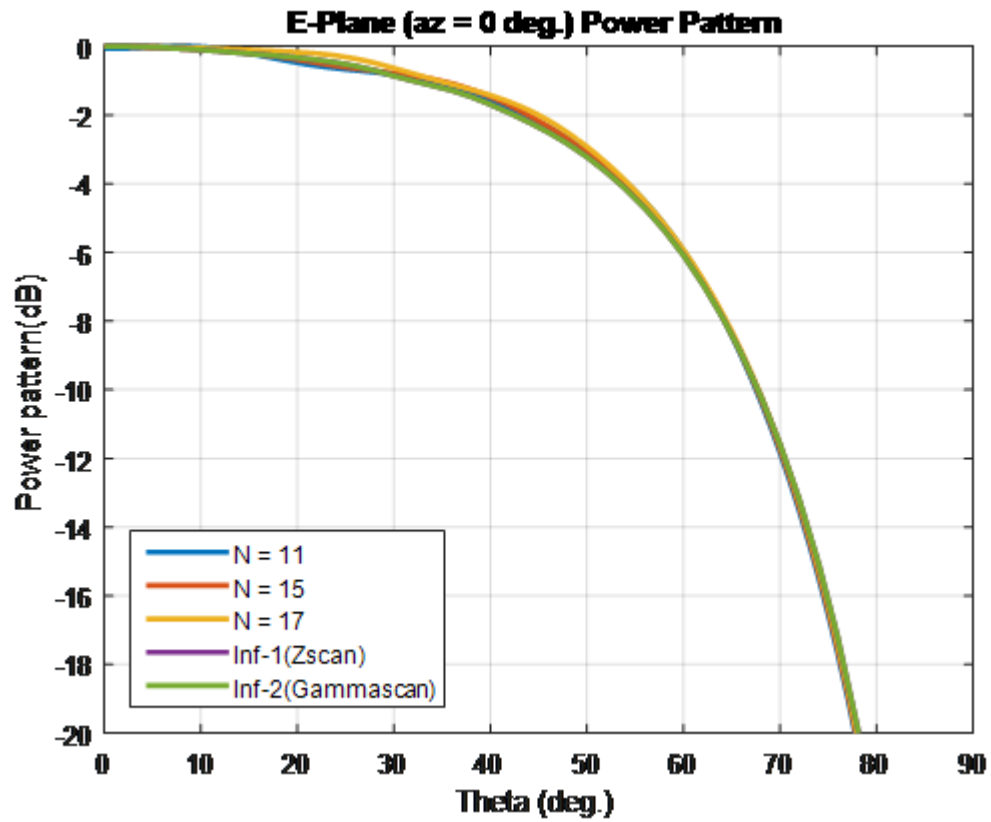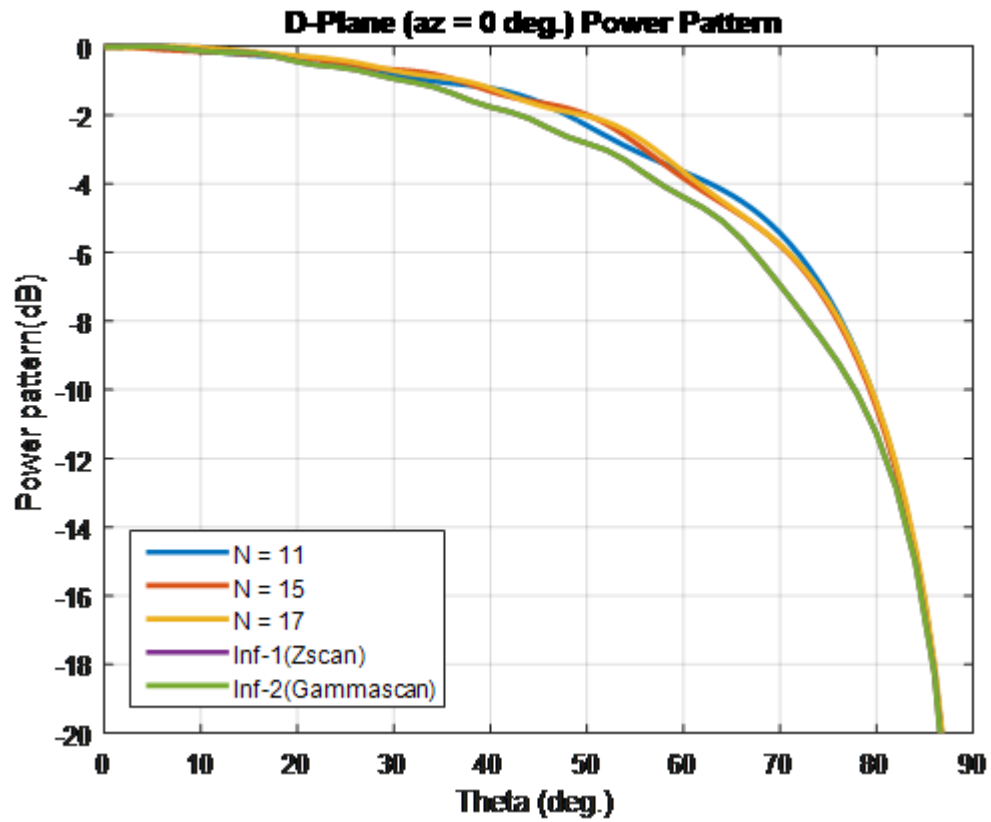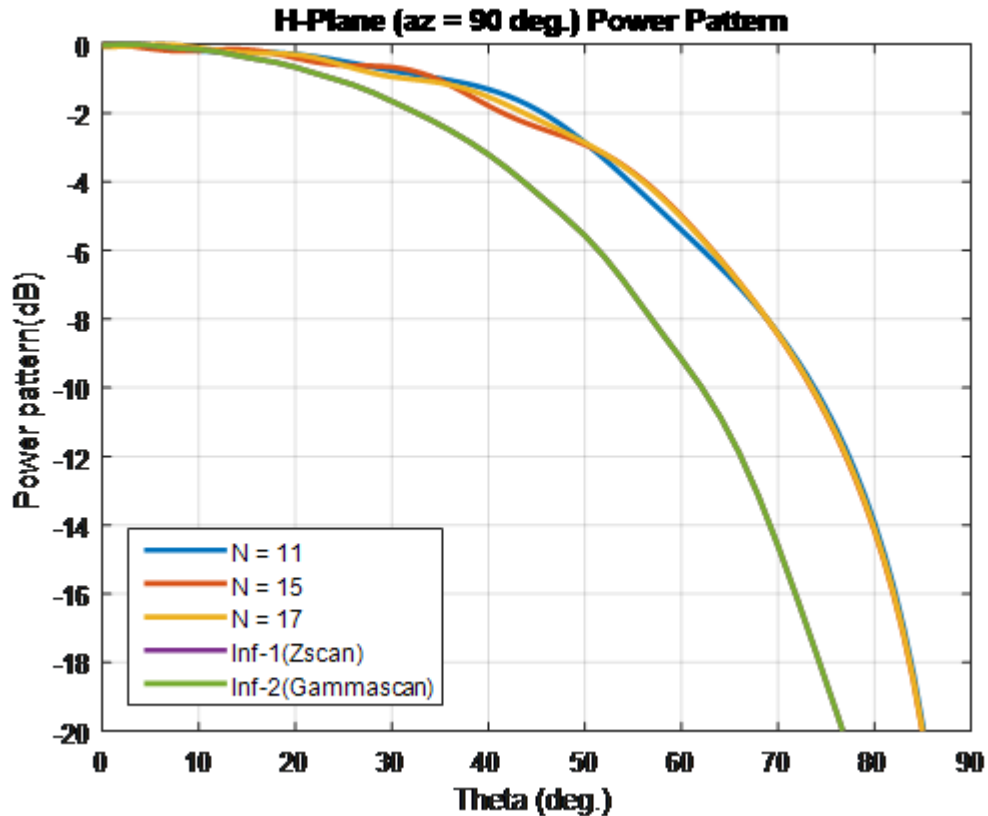
Case 2: Compare finite array and infinite array with unit cell of dimensions $0.7\lambda \times 0.7\lambda$

To compare the scan element pattern of these array types and infinite arrays, repeat the process in case 1. Using these unit cell dimensions creates grating lobes. Terminate the finite arrays using 86-Ω resistance. For an infinite array with unit cell of dimensions $0.7\lambda \times 0.7\lambda$, the broadside resistance is 86 Ω.

E-Plane (az = 0 deg.) Power Pattern

D-Plane (az = 0 deg.) Power Pattern
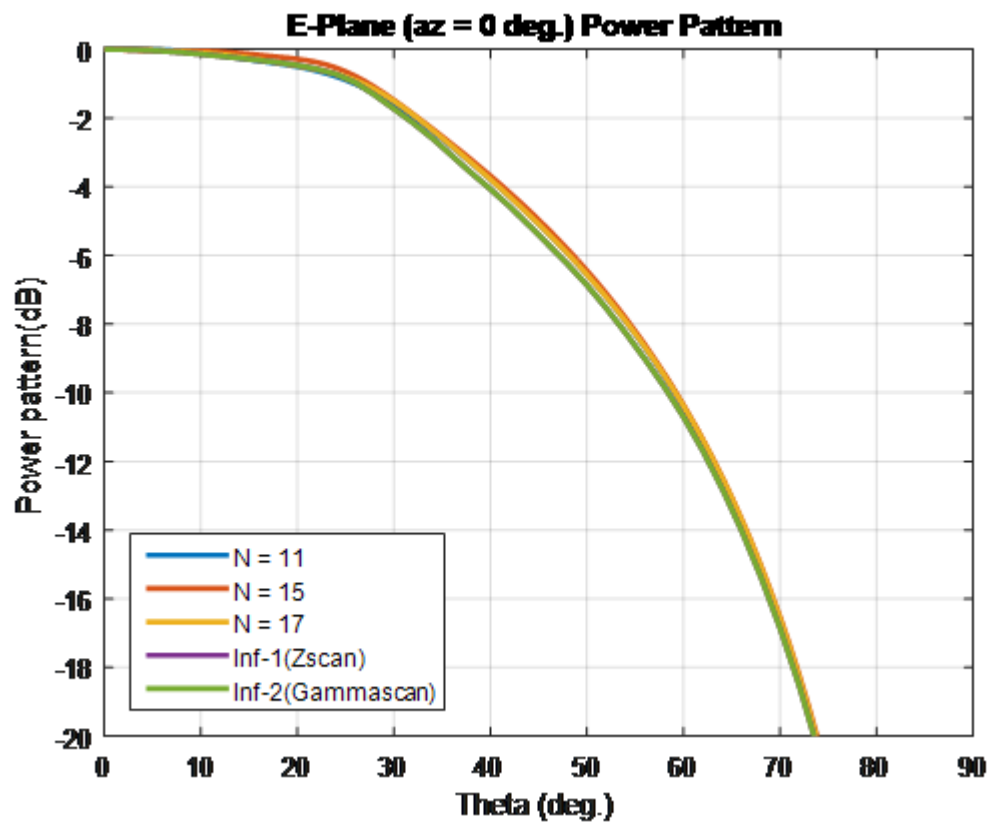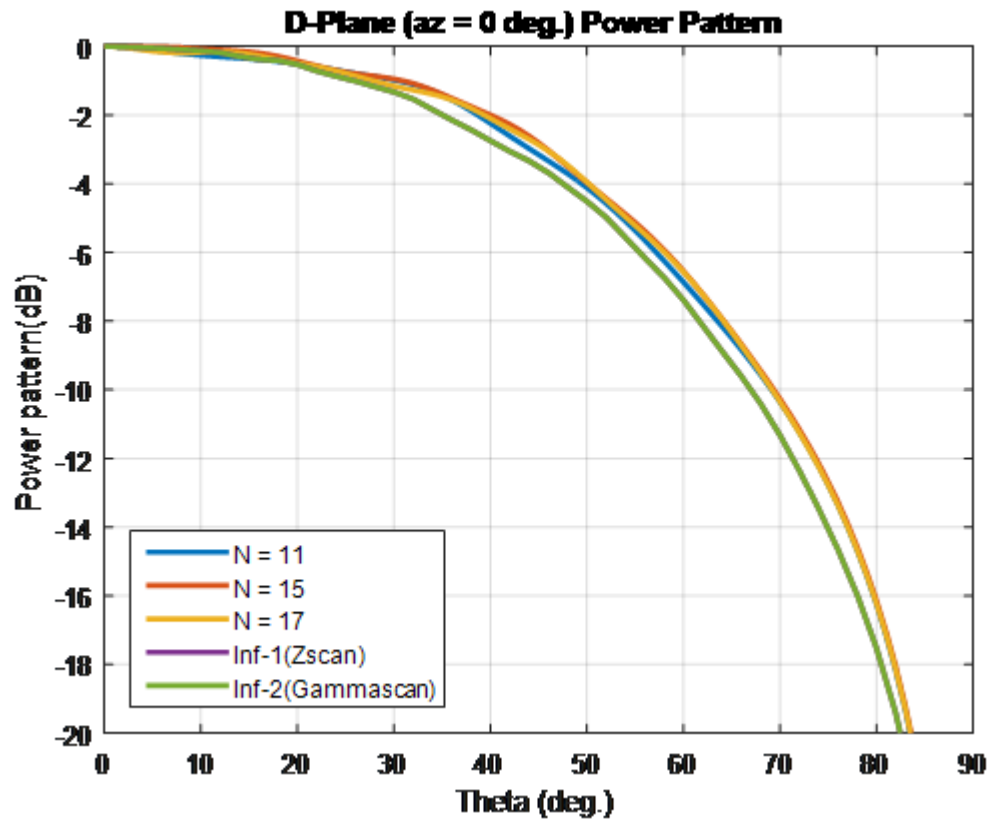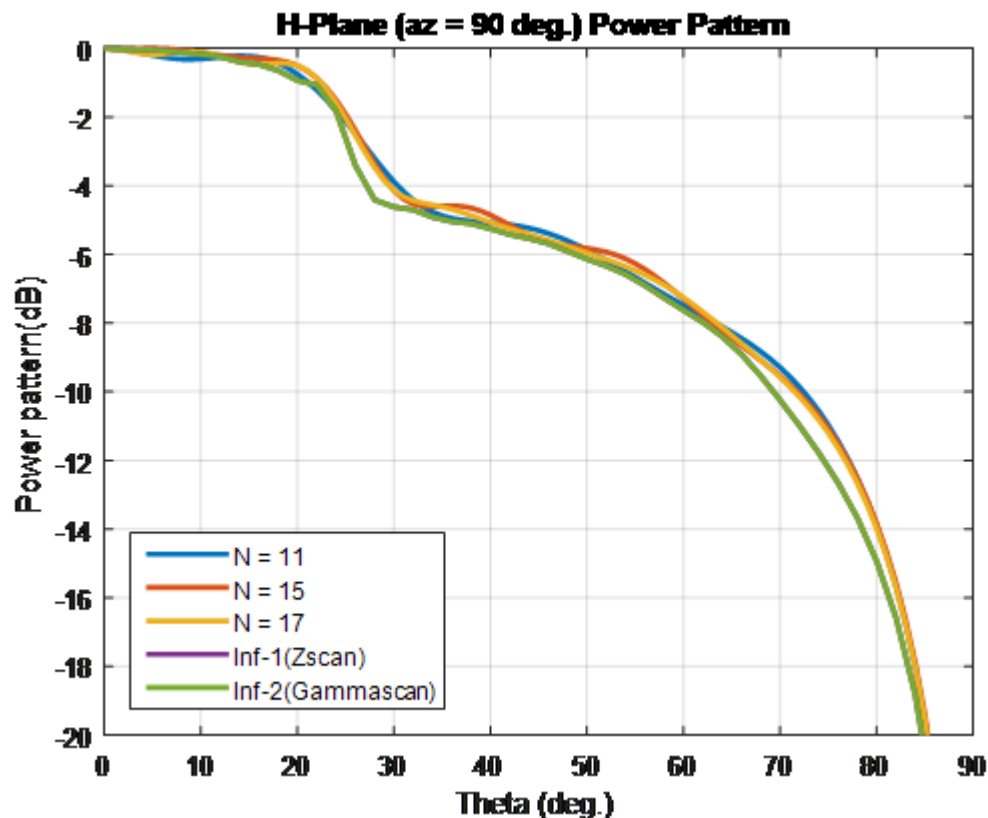
For finite arrays of size greater than 10 x 10, the scan element patterns in the E-, D-, and H-planes match the patterns of the infinite array scan element.

Impact of Infinite Double Summation

As show in the Green's equations, the periodic Green's function has an infinite double summation in (m, n). When performing infinite array analysis, the number of terms in the double summation affects the accuracy of the final solution. Higher number of terms results in better accuracy but increases computation time.

By default, Antenna Toolbox uses 10 terms for each summation term (m, n) to perform infinite array analysis. The total summation term length is 2*10+1 (-10 to +10). To modify the number of terms, use the method `numSummationTerms`.

Higher number of terms are required if:

• You observe negative values for scan resistance for certain scan angles at certain frequencies.
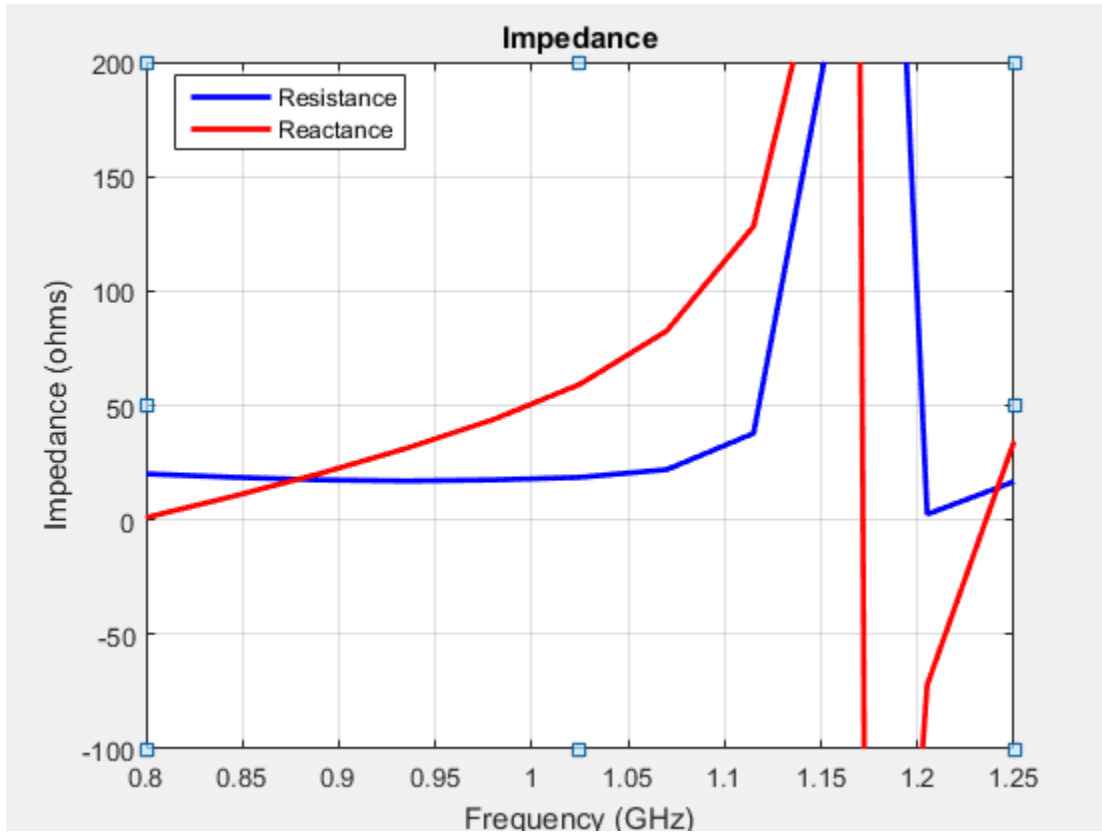
• You must investigate for convergence when scan impedance shows slow variations.
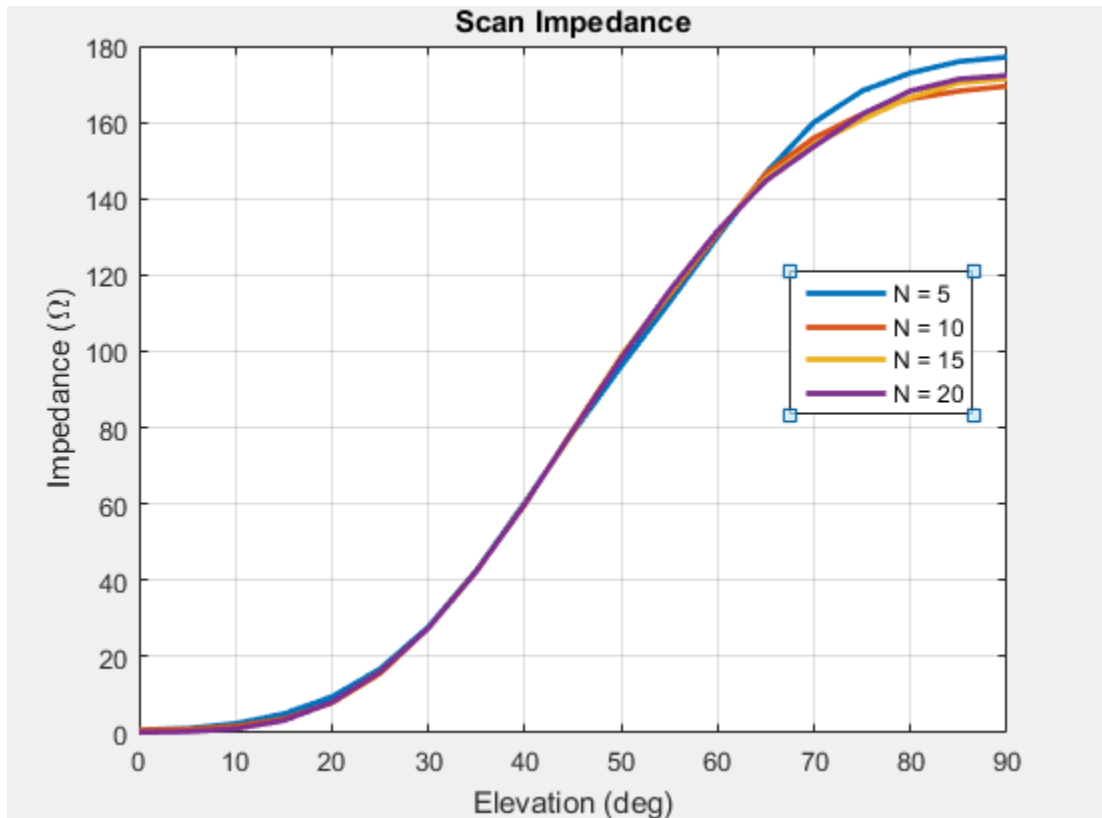
References

[1] Mailloux, R. J. *Phased Array Antenna Handbook*. Norwood, MA: Artech House. 2nd Edition. 2005.

[2] Hansen, R. C. *Phased Array Antennas*. Hoboken, NJ: John Wiley & Sons Inc. 2nd Edition. 1998, pp.221–313.

[3] Allen, J. "Gain and impedance variation in scanned dipole arrays." *IRE Transactions on Antennas and Propagation*. Vol. 10, Number 5, September 1962, pp.566–572.

[4] Wasylkiwskyj, W., and W. Kahn. "Efficiency as a measure of size of a phased-array antenna." *IEEE Transactions on Antennas and Propagation*. Vol.21, Number 6, November 1973, pp.879–884.

[5] Holter, H., and H. Steyskal. "On the size requirement for finite phased-array models." *IEEE Transactions on Antennas and Propagation*. Vol.50, Number 6, June 2002, pp.836–840.

Manipulate Array Elements

This example shows you how to control each individual element in a linear or rectangular array. You can use this technique to change the size and tilt of the antenna, or to model dead elements etc. with individual elements in an array.

Define Dipole Array

Create a dipole antenna using the `dipole` class. To create a 5x5 dipole array, replicate the dipole antenna using a 5x5 matrix. Create a rectangular array using the dipole array as a single element.

```
d = dipole;
N = 5;
df = repmat(d,N)
r = rectangularArray('Element',df);
show(r)


df =

  5×5 dipole array with properties:

        Length: {25×1 cell}
         Width: {25×1 cell}
    FeedOffset: {25×1 cell}
          Tilt: {25×1 cell}
      TiltAxis: {25×1 cell}
          Load: {25×1 cell}
```

rectangularArray of dipole antennas
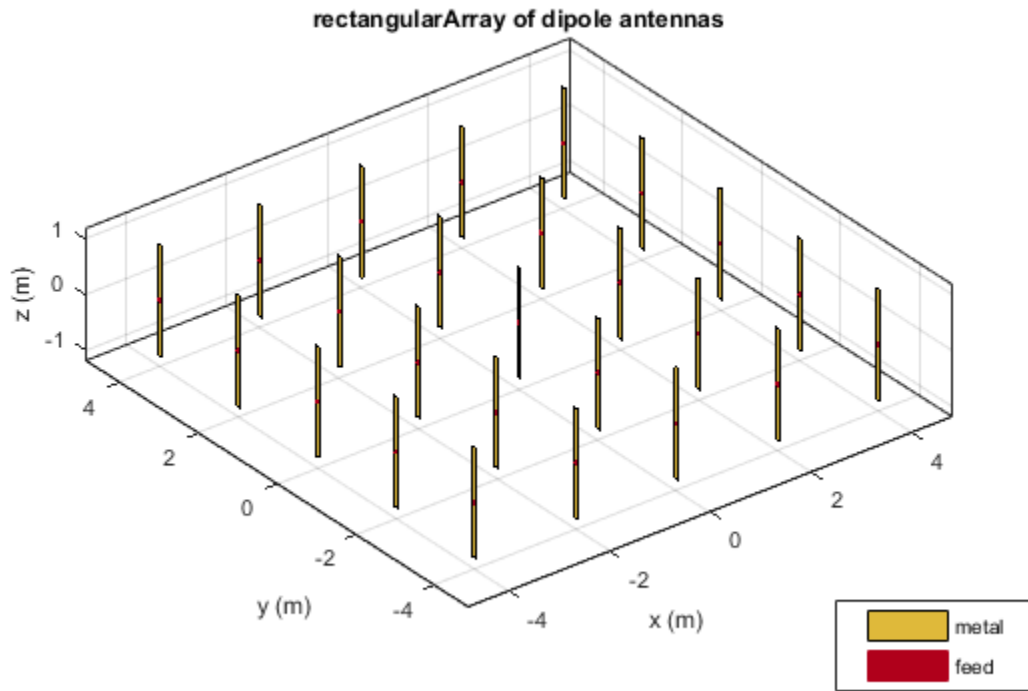
Tilt Alternate Elements

Create a vector of alternate elements in the rectangular array.

```
S = [1:2:N*N];
```

Tilt the alternate elements in the array by 90 degrees about the y-axis.

```
for i = 1:25
    if any(S==i)
      r.Element(i).Tilt = 90;
   r.Element(i).TiltAxis = [0 1 0];
    end
```

```
end
show(r)
```

rectangularArray of dipole antennas
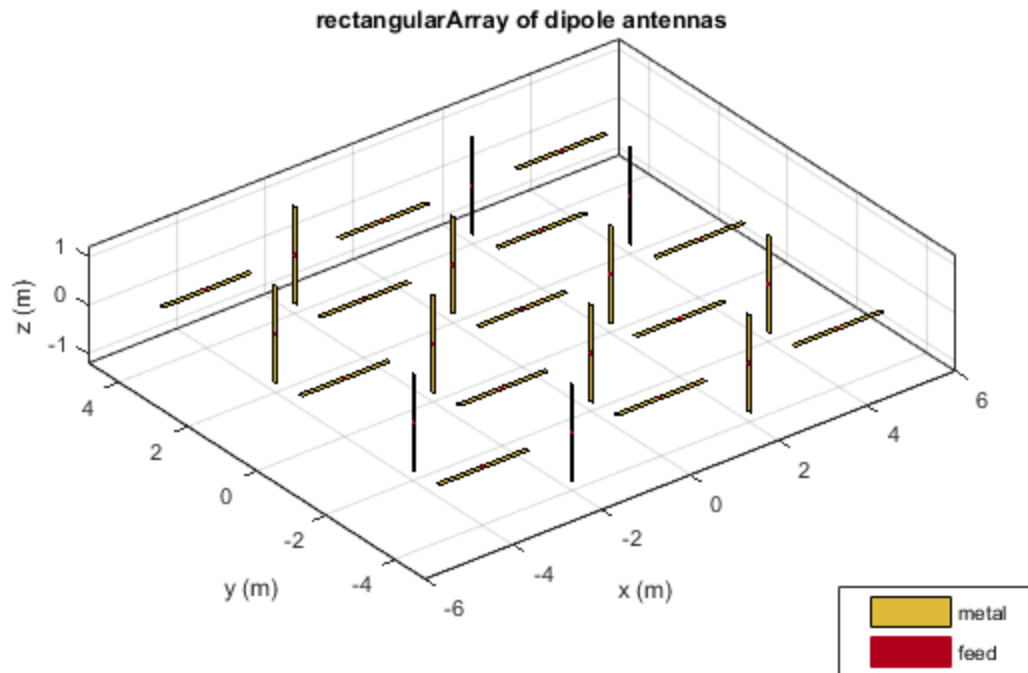
Pattern of Rectangular Array

Plot the pattern of the array at 75 MHz.

```
pattern(r,75e6)
```
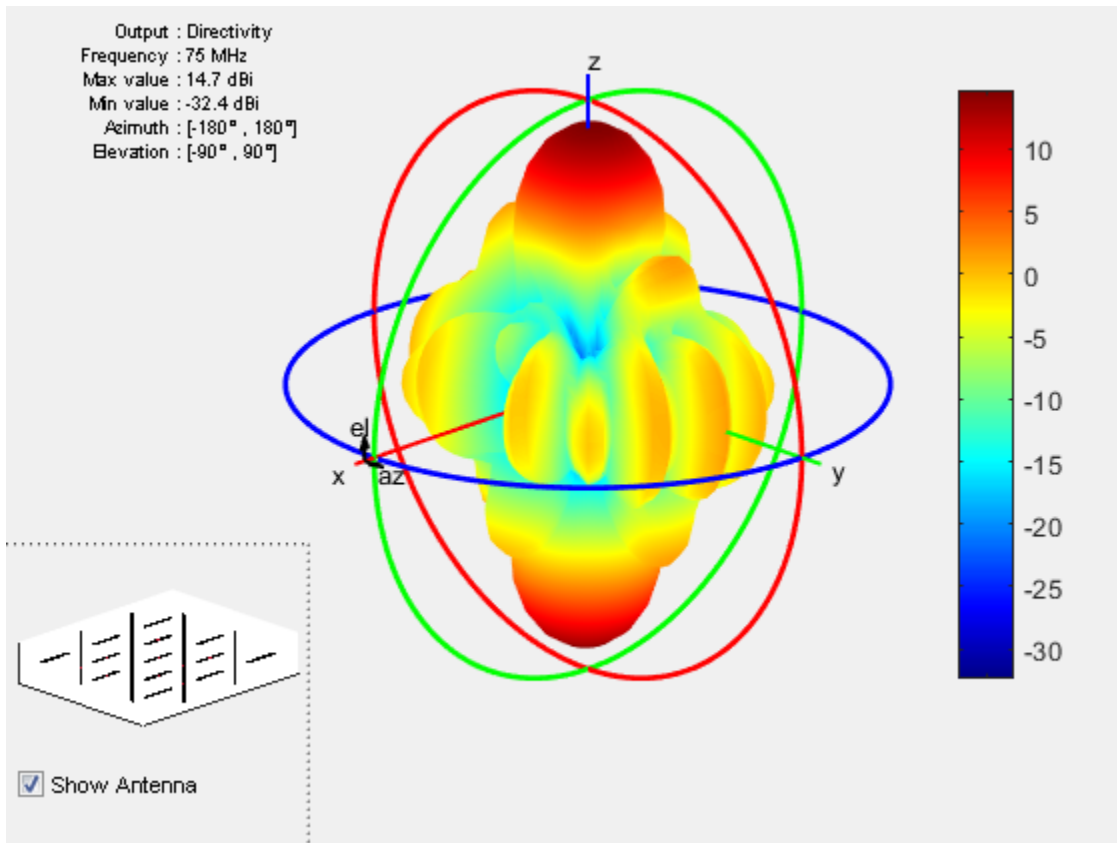
Model Dead Elements

Antennas with a zero excitation voltage feed are called dead elements. By default, each element in an array is excited by an amplitude of 1 V.

```
Vfeed = ones(1,N*N)


Vfeed =

  Columns 1 through 13

    1    1    1    1    1    1    1    1    1    1    1    1    1
```

```
   Columns 14 through 25

    1     1     1     1     1     1     1     1     1     1     1     1
```

To model dead elements, set the voltage to zero for all horizontal elements. To control voltage, use the `AmplitudeTaper` property. This property is the excitation amplitude of the antennas in an array.

```
Vfeed(S) = 0
r.AmplitudeTaper = Vfeed


Vfeed =

  Columns 1 through 13

    0     1     0     1     0     1     0     1     0     1     0     1     0

  Columns 14 through 25

    1     0     1     0     1     0     1     0     1     0     1     0


r =

  rectangularArray with properties:

          Element: [5×5 dipole]
             Size: [5 5]
       RowSpacing: 2
    ColumnSpacing: 2
          Lattice: 'Rectangular'
    AmplitudeTaper: [0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0]
       PhaseShift: 0
             Tilt: 0
          TiltAxis: [1 0 0]
```

Plot the radiation pattern of the array.

```
figure
pattern(r,75e6)
```

Output : Directivity
Frequency : 75 MHz
Max value : 6.87 dBi
Min value : -46.4 dBi
Azimuth : [-180° , 180°]
Elevation : [-90° , 90°]

☑ Show Antenna
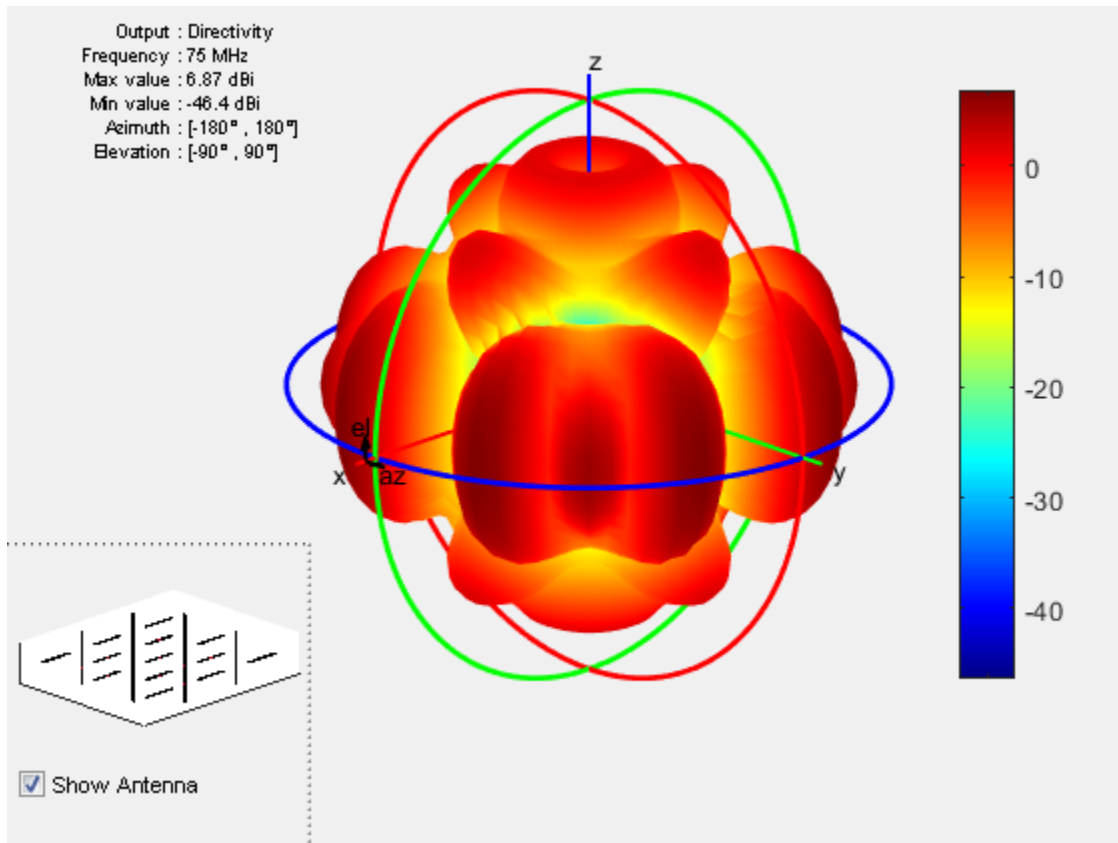
Computational Techniques

Method of Moments – Metal

Method of Moments computation technique for metal antennas.

The first step in the computational solution of electromagnetic problems is to discretize Maxwell's equations. The process results in this matrix-vector system:

$$V = ZI$$

- V — Applied voltage vector. This signal can be voltage or power applied to the antenna or an incident signal falling on the antenna.
- I — Current vector that represents current on the antenna surface.
- Z — Interaction matrix or impedance matrix that relates V to I.

Antenna Toolbox uses method of moments (MoM) to calculate the interaction matrix and solve system equations.

MoM Formulation

The MoM formulation is split into three parts.

Discretization of Metals

Discretization enables the formulation from the continuous domain to the discrete domain. This step is called *meshing* in antenna literature. In the MoM formulation, the metal surface of the antenna is meshed into triangles.
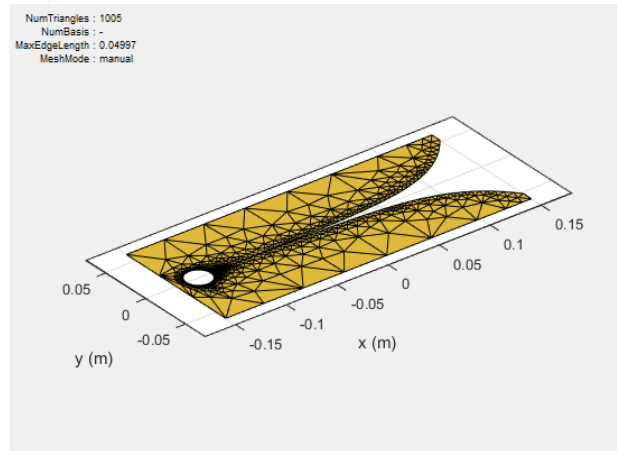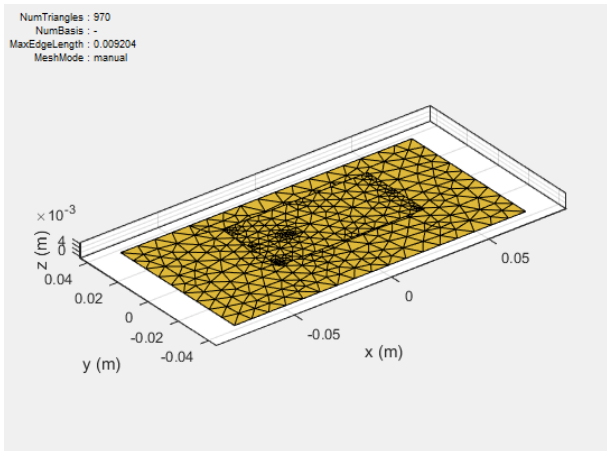
Basis Functions

To calculate the surface currents on the antenna structure, you first define basis functions. Antenna Toolbox uses Rao-Wilton-Glisson (RWG) [2] basis functions. The arrows show the direction of current flow.
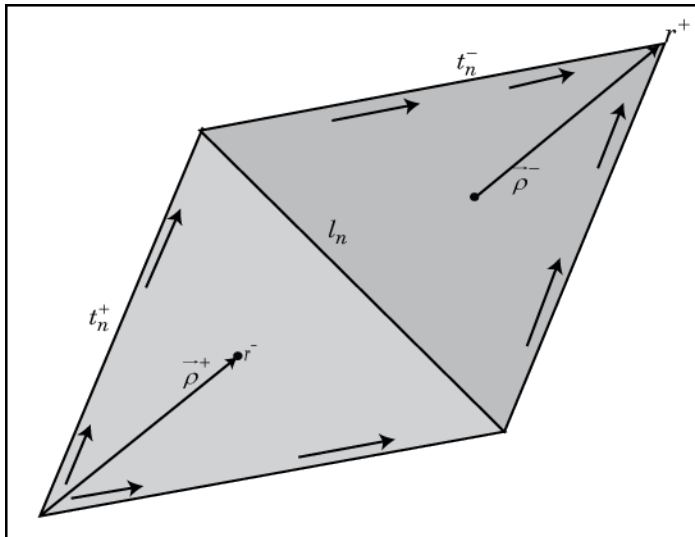
The basis function includes a pair of adjacent (not necessarily coplanar) triangles and resembles a small spatial dipole with linear current distribution. Each triangle is associated with a positive or negative charge.

For any two triangle patches, t_n^+ and t_n^-, having areas A_n^+ and A_n^-, and sharing common edge l_n, the basis function is

$$\vec{f}_n(\vec{r}) = \begin{cases} \dfrac{l_n}{2A_n^+} \vec{\rho}_n^{+S}, & \vec{r} \in \mathrm{t}_n^+ \\[2mm] \dfrac{l_n}{2A_n^-} \vec{\rho}_n^{-S}, & \vec{r} \in \mathrm{t}_n^- \end{cases}$$

- $\vec{\rho}_n^+ = \vec{r} - \vec{r}_n^+$ — Vector drawn from the free vertex of triangle t_n^+ to observation point $\vec{r}$

- $\vec{\rho}_n^- = \vec{r}_n^+ - \vec{r}$ — Vector drawn from the observation point to the free vertex of the triangle t_n^-

and

$$\nabla \cdot \vec{f}_n(\vec{r}) = \begin{cases} \dfrac{l_n}{A_n^+}, & \vec{r} \in \mathrm{t}_n^+ \\[2mm] -\dfrac{l_n}{A_n^-}, & \vec{r} \in \mathrm{t}_n^- \end{cases}$$

The basis function is zero outside the two adjacent triangles t_n^+ and t_n^-. The RWG vector basis function is linear and has no flux (no normal component) through its boundary.

Interaction Matrix

The interaction matrix is a complex dense symmetric matrix. It is a square N-by-N matrix, where N is the number of basis functions, that is, the number of interior edges in the structure. A typical interaction matrix for a structure with 256 basis functions is shown:
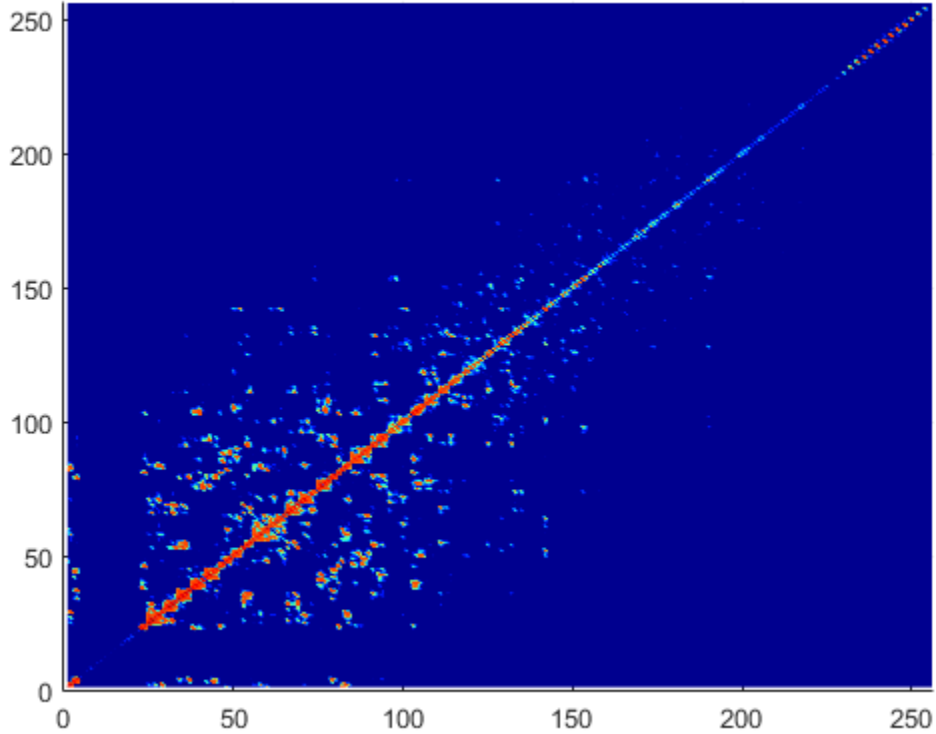
To fill out the interaction matrix, calculate the free-space Green's function between all basis functions on the antenna surface. The final interaction matrix equations are:

$$Z_{mn} = \left(\frac{j\omega\mu}{4\pi} \right) \iint_{SS} \vec{f}_m\left(\vec{r}\right).\vec{f}_m\left(\vec{r}'\right) g d\vec{r}' d\vec{r} - \left(\frac{j}{4\pi\omega\varepsilon} \right) \iint_{SS} \left(\nabla.\vec{f}_m\right)\left(\nabla.\vec{f}_m\right) g d\vec{r}' d\vec{r}$$

where

- $g(\vec{r},\vec{r}') = \dfrac{\exp(-jk\left|\vec{r}-\vec{r}'\right|)}{\left|\vec{r}-\vec{r}'\right|}$ — Free-space Green's function

To calculate the interaction matrix, excite the antenna by a voltage of 1 V at the feeding edge. So the voltage vector has zero values everywhere except at the feeding edge. Solve the system of equations to calculate the unknown currents. Once you determine the unknown currents, you can calculate the field and surface properties of the antenna.

Neighbor Region

From the interaction matrix plot, you observe that the matrix is diagonally dominant. As you move further away from the diagonal, the magnitude of the terms decreases. This behavior is same as the Green's function behavior. The Green's function decreases as the distance between r and r' increases. Therefore, it is important to calculate the region on the diagonal and close to the diagonal accurately.

This region on and around the diagonal is called *neighbor region*. The neighbor region is defined within a sphere of radius R, where R is in terms of triangle size. The size of a triangle is the maximum distance from the center of the triangle to any of its vertices. By default, R is twice the size of the triangle. For better accuracy, a higher-order integration scheme is used to calculate the integrals.
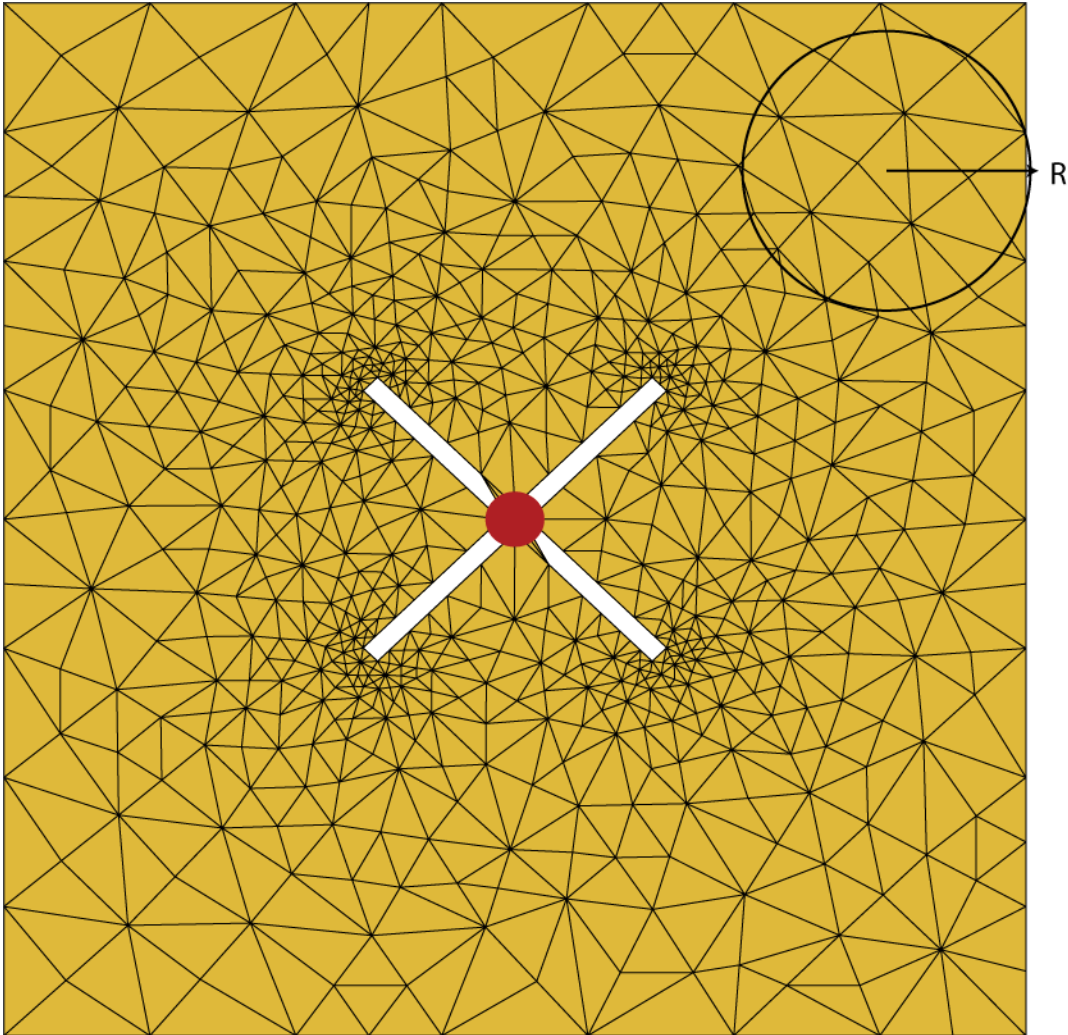
Singularity Extraction

Along the diagonal, r and r' are equal and the defines Green's function becomes singular. To remove the singularity, extraction is performed on these terms.

$$\iint_{t_p \, t_q} \left(\vec{\rho}_i \cdot \vec{\rho}_j'\right) g(\vec{r}, \vec{r}') ds' ds = \iint_{t_p \, t_q} \frac{\left(\vec{\rho}_i \cdot \vec{\rho}_j'\right)}{\left|\vec{r} - \vec{r}'\right|} ds' ds + \iint_{t_p \, t_q} \frac{\left(\exp\left(-jk\left|\vec{r} - \vec{r}'\right|\right) - 1\right)\left(\vec{\rho}_i \cdot \vec{\rho}_j'\right)}{\left|\vec{r} - \vec{r}'\right|} ds' ds$$

$$\iint_{t_p \, t_q} g(\vec{r}, \vec{r}') ds' ds = \iint_{t_p \, t_q} \frac{1}{\left|\vec{r} - \vec{r}'\right|} ds' ds + \iint_{t_p \, t_q} \frac{\left(\exp\left(-jk\left|\vec{r} - \vec{r}'\right|\right) - 1\right)}{\left|\vec{r} - \vec{r}'\right|} ds' ds$$

The two integrals on the right-hand side of the equations, called potential or static integrals are found using analytical results [3].

Finite Arrays

The MoM formulation for finite arrays is the same as for a single antenna element. The main difference is the number of excitations (feeds). For finite arrays, the voltage vector is now a voltage matrix. The number of columns are equal to the number of elements in the array.
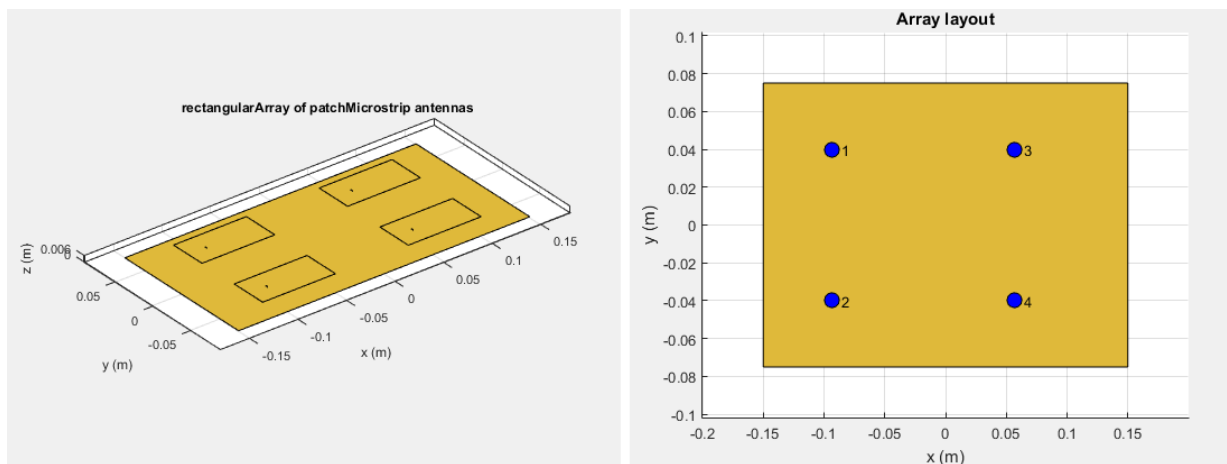
For example, the voltage vector matrix for a **2x2** array of rectangular patch antenna has four columns as each antenna can be excited separately.

Infinite Array

To model an infinite array, you change the MoM to account for the infinite behavior. To do so you replace the free-space Green's functions with periodic Green's functions. The periodic Green's function is an infinite double summation.

Green's Function	Periodic Green's Function
$$g = \frac{e^{-jkR}}{R}$$ $$R = \lvert \vec{r} - \vec{r}' \rvert$$	$$g_{\text{periodic}} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{j\phi_{mn}} \frac{e^{-jkR_{mn}}}{R_{mn}}$$ $$R_{mn} = \sqrt{\left(x - x' - x_m\right)^2 + \left(y - y' - y_n\right)^2 + \left(z - z'\right)^2}$$ $$\phi_{mn} = k\left(x_m \sin\theta \cos\varphi + y_n \sin\theta \cos\varphi\right)$$ $$x_m = m \cdot d_x, \qquad y_n = n \cdot d_y$$

d_x and d_y are the ground plane dimensions that define the x and y dimensions of the unit cell. θ and Φ are the scan angles.

Comparing the two Green's functions, you observe an additional exponential term that is added to the infinite sum. The Φ_{mn} accounts for the scanning of the infinite array. The periodic Green's function also accounts for the effect of mutual coupling.

For more information see, "Infinite Arrays" on page 2-20.

References

[1] Harringhton, R. F. *Field Computation by Moment Methods*. New York: Macmillan, 1968.

[2] Rao, S. M., D. R. Wilton, and A. W. Glisson. "Electromagnetic scattering by surfaces of arbitary shape." *IEEE. Trans. Antennas and Propagation*, Vol. AP-30, No. 3, May 1982, pp. 409–418.

[3] Wilton, D. R., S. M. Rao, A. W. Glisson, D. H. Schaubert, O. M. Al-Bundak. and C. M. Butler. "Potential Integrals for uniform and linear source distribution on polygonal and polyhedral domains." *IEEE. Trans. Antennas and Propagation*. Vol. AP-30, No. 3, May 1984, pp. 276–281.

[4] Balanis, C.A. *Antenna Theory. Analysis and Design*. 3rd Ed. New York: John Wiley & Sons, 2005.

More About

Method of Moments – Metal and Dielectric

Method of Moments computation technique for metal and dielectric antennas.

In this section...

Antennas using dielectric substrate consists of a metal part and a dielectric part. The first step in the computational solution of electromagnetic problems is to discretize Maxwell's equations. The process results in this matrix-vector system:

$$V = ZI$$

- V — Applied voltage vector. This signal can be voltage or power applied to the antenna or an incident signal falling on the antenna.
- I — Current vector that represents current on the antenna surface.
- Z — Interaction matrix or impedance matrix that relates V to I. For calculating the interaction matrix, the effect of metal and dielectric parts in an antenna are taken separately.

Antenna Toolbox uses method of moments (MoM) to calculate the interaction matrix and solve system equations.

MoM Formulation

The MoM formulation is split into three parts.

Discretization of Dielectrics

Discretization enables the formulation from the continuous domain to the discrete domain. This step is called *meshing* in antenna literature. In the MoM formulation,

the metal surface of the antenna is meshed into triangles and the dielectric volume is meshed into tetrahedrons.
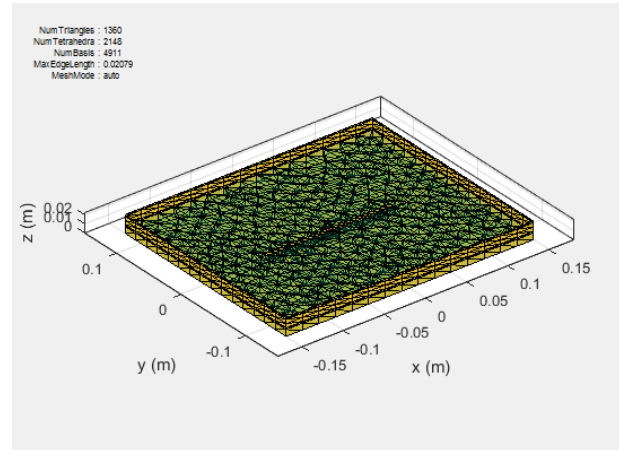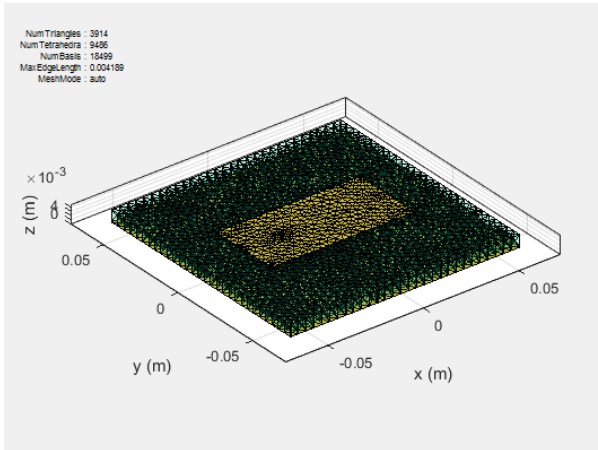
Basis Functions

Basis functions are used to represent unknown quantities. In the case of antennas using dielectrics, the unknown quantities are the surface current on the metal structure and flux density due to dielectric volume. Antenna Toolbox uses Rao-Wilton-Glisson (RWG) [2] basis functions. For basis functions for a metal structure in an antenna refer, "Method of Moments – Metal" on page 3-2.

For the dielectric volume of the antenna, Antenna Toolbox uses a zeroth order edge basis function to model the flux density.
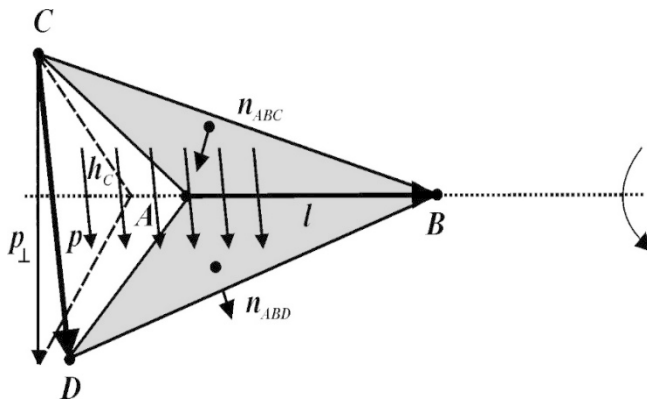
The figure shows an edge-based basis function. The vector variation is perpendicular to the base edge AB (or $\bar{l}$). The vector of the edge CD (or $\bar{p}$) defines the basis function. Within a tetrahedron, the basis function is a constant field given by

$$\vec{f} = c\vec{p}$$

- c – normalization coefficient.
- p – vector of the edge defining the basis function.

Interaction Matrix

The interaction matrix is a complex dense symmetric matrix. For a metal-dielectric antenna, there are two sets of basis functions and four interactions. To fill out the interaction matrix, calculate the free-space Green's function between all the basis functions on the antenna surface. The final interaction matrix equations are:

- Z_{MM} – metal to metal interaction. For a pure metal structure, you only calculate this symmetric square matrix.

$$Z_{mn}^{MM} = \left(\frac{j\omega\mu}{4\pi}\right)\iint_{S}\int_{S}\vec{f}_m^M(\vec{r}).\vec{f}_n^M(\vec{r}')g d\vec{r}'d\vec{r} - \left(\frac{j}{4\pi\omega\varepsilon}\right)\iint_{S}\int_{S}\left(\nabla.\vec{f}_m^M\right)\left(\nabla.\vec{f}_n^M\right)g d\vec{r}'d\vec{r}$$

- Z_{DD} – dielectric to dielectric interaction. For pure dielectric structures, you only calculate this symmetric square matrix.

$$\hat{Z}_{mn}^{DD} = \sum_{p-1}^{P}\sum_{p'-1}^{P'}\frac{K_p}{\hat{\varepsilon}_p}\int_{V_D}\vec{f}_{mp}(\vec{r})\cdot\vec{f}_{np'}(\vec{r})d\vec{r}$$
$$- \frac{\omega^2\mu_0}{4\pi}\sum_{p-1}^{P}\sum_{p'-1}^{P'}K_p K_{p'}\int_{V_D}\int_{V_{D'}}g(\vec{r},\vec{r}')\vec{f}_{mp}(\vec{r})\cdot\vec{f}_{np'}(\vec{r}')d\vec{r}d\vec{r}'$$
$$- \frac{1}{4\pi\varepsilon_0}\sum_{q-1}^{Q}\sum_{q'-1}^{Q'}\hat{K}_q\hat{K}_{q'}\int_{\Omega_q}\int_{\Omega_{q'}}g(\vec{r},\vec{r}')f_{\perp mq}(\vec{r})f_{\perp nq'}(\vec{r}')d_s d_{s'} \qquad m,n = 1,....,N$$

- Z_{MD} and Z_{DM} – These matrices calculate the interaction between metal and dielectric. This matrix is not a symmetrical square matrix.

$$Z_{mn}^{MD} = -\frac{\omega^2 \mu_0}{4\pi} \sum_{p-1}^{2} \sum_{p'-1}^{P'} K_p \int_t \int_{V_{D'}} \vec{f}_n^M(\vec{r}) \cdot \vec{f}_{mp'}(\vec{r}') g(\vec{r},\vec{r}') d\vec{r}' d_s$$

$$-\frac{1}{4\pi\varepsilon_0} \sum_{p-1}^{2} \sum_{q-1}^{Q} K_q \int_{t_D} \int_{\Omega_{q'}} (\nabla_s \cdot \vec{f}_n^M(\vec{r})) f_{\perp mq}(\vec{r}') g(\vec{r},\vec{r}') d_{\Omega'} d_s$$

$$m = 1,....,N_D; n = 1,...,N_M$$

$$Z_{mn}^{DM} = -\frac{j\omega\mu_0}{4\pi} \sum_{p-1}^{2} \sum_{p'-1}^{P'} K_{p'} \int_{V_D} \int_{S_{D'}} \vec{f}_{np}(\vec{r}) \cdot \vec{f}_m^M(\vec{r}') g(\vec{r},\vec{r}') d_{s'} d\vec{r}$$

$$+\frac{1}{4\pi\varepsilon_0\omega} \sum_{p-1}^{2} \sum_{q-1}^{Q} \hat{K}_q \int_{\Omega_q} \int_{S_{D'}} f_{\perp nq}(\vec{r}) \cdot (\nabla_s \cdot \vec{f}_m^M(\vec{r}')) g(\vec{r},\vec{r}') d_{s'} d_{\Omega}$$

$$m = 1,....,N_D; n = 1,...,N_M$$

where

- $$g(\vec{r},\vec{r}') = \frac{\exp(-jkR)}{R}, R = |\vec{r} - \vec{r}'|$$ is the free space Greens's function.

- $$K = \frac{\hat{\varepsilon}^{\pm} - \varepsilon_0}{\hat{\varepsilon}^{\pm}}$$ is the complex dielectric constant within every tetrahedron.

- $$\hat{K}_q = K_+ - K_-$$ is the differential contrast on every face of the tetrahedron.

For a composite metal structure, you must calculate all four matrices.

Neighbor Region

The figure shows a typical interaction matrix for a metal structure Z_{MM} with 256 basis functions.
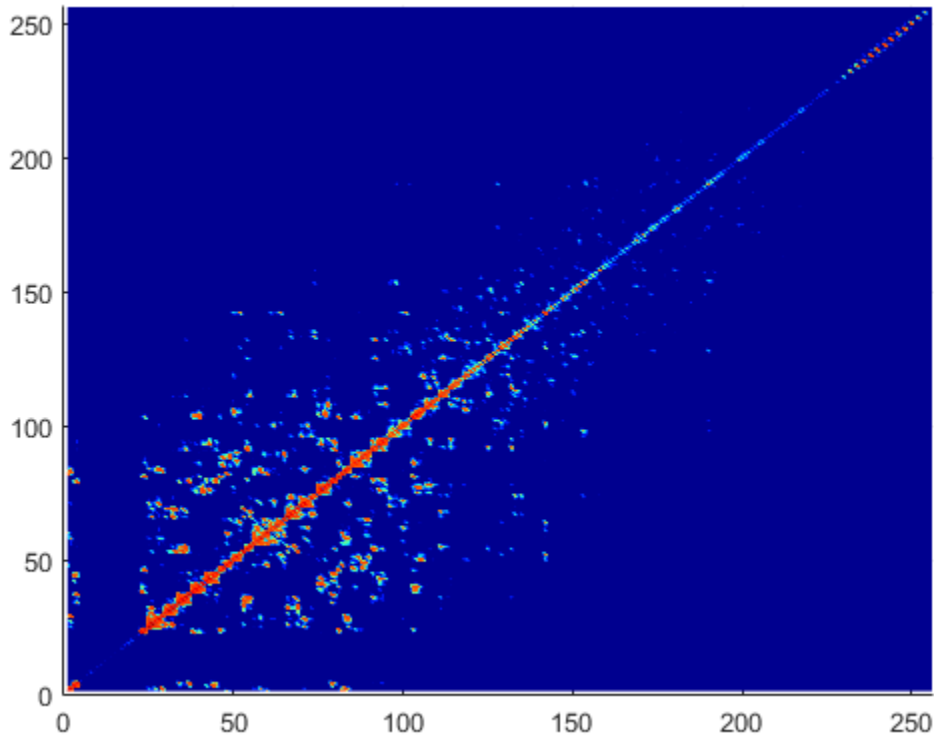
From the interaction matrix plot, you observe that the matrix is diagonally dominant. The dielectric interaction matrix is also diagonally dominant. As you move further away from the diagonal, the magnitude of the terms decreases. This behavior is same as the Green's function behavior. The Green's function decreases as the distance between r and r' increases. Therefore, it is important to calculate the region on the diagonal and close to the diagonal accurately.

This region on and around the diagonal is called *neighbor region*. For a metal-dielectric antenna, the neighborhood region is based on the average size of the tetrahedron.

For neighboring region details for metal antennas refer, "Method of Moments – Metal" on page 3-2.

Singularity Extraction

Along the diagonal, *r* and *r'* are identical and the defined Green's function becomes singular. To remove the singularity, extraction is performed on these terms. The equations for the singularity extraction of the Z_{MM} matrix are:

$$\iint_{t_p\,t_q} \left(\vec{\rho}_i\cdot\vec{\rho}'_j\right)g(\vec{r},\vec{r}')ds'ds = \iint_{t_p\,t_q} \frac{\left(\vec{\rho}_i\cdot\vec{\rho}'_j\right)}{|\vec{r}-\vec{r}'|}ds'ds + \iint_{t_p\,t_q} \frac{\left(\exp\left(-jk|\vec{r}-\vec{r}'|\right)-1\right)\left(\vec{\rho}_i\cdot\vec{\rho}'_j\right)}{|\vec{r}-\vec{r}'|}ds'ds$$

$$\iint_{t_p\,t_q} g(\vec{r},\vec{r}')ds'ds = \iint_{t_p\,t_q} \frac{1}{|\vec{r}-\vec{r}'|}ds'ds + \iint_{t_p\,t_q} \frac{\left(\exp\left(-jk|\vec{r}-\vec{r}'|\right)-1\right)}{|\vec{r}-\vec{r}'|}ds'ds$$

The two integrals on the right-hand side of the equations, called potential or static integrals are found using analytical results [3].

The equations for the singularity extraction of the Z_{DD} matrix are:

$$\iint_{V_D\,V_{D'}} g(\vec{r},\vec{r}')d\vec{r}d\vec{r}' = \iint_{V_D\,V_q} \frac{1}{|\vec{r}-\vec{r}'|}d\vec{r}d\vec{r}' + \iint_{V_D\,V_q} \frac{\left(\exp\left(-jk|\vec{r}-\vec{r}'|\right)-1\right)}{|\vec{r}-\vec{r}'|}d\vec{r}d\vec{r}'$$

$$\iint_{\Omega_q\,\Omega_{q'}} g(\vec{r},\vec{r}')d\Omega d\Omega' = \iint_{\Omega_D\,\Omega_q} \frac{1}{|\vec{r}-\vec{r}'|}d\Omega d\Omega' + \iint_{S_D\,S_q} \frac{\left(\exp\left(-jk|\vec{r}-\vec{r}'|\right)-1\right)}{|\vec{r}-\vec{r}'|}d\Omega d\Omega$$

Finite Arrays

The MoM formulation for finite arrays is the same as for a single antenna element. The main difference is the number of excitations (feeds). For finite arrays, the voltage vector is now a voltage matrix. The number of columns are equal to the number of elements in the array.
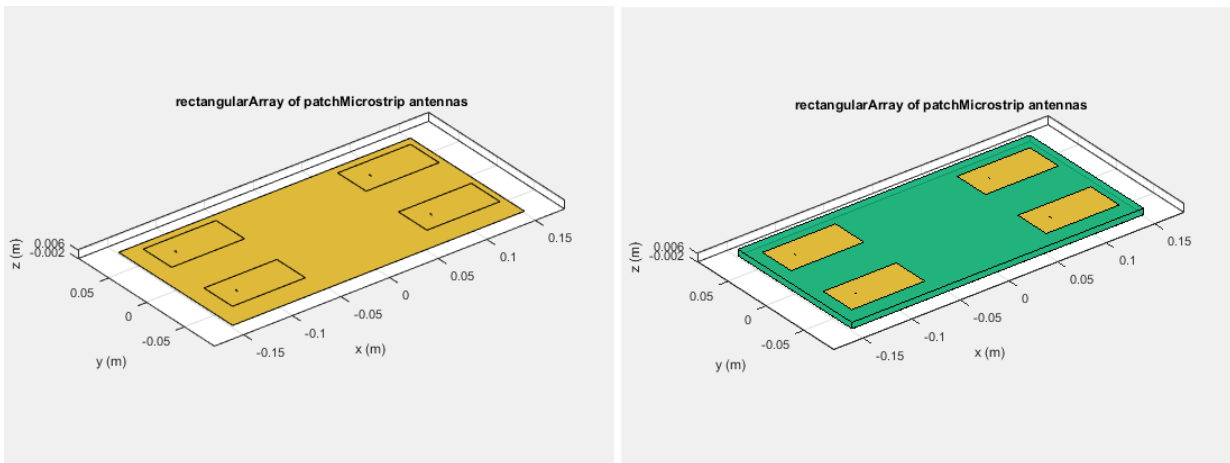
For example, the voltage vector matrix for a 2x2 array of rectangular patch antenna (with and without dielectric substrate) has four columns as each antenna can be excited separately.

Infinite Array

To model an infinite array, you change the MoM to account for the infinite behavior. To do so, you replace the free-space Green's functions with periodic Green's functions. The periodic Green's function is an infinite double summation.

Green's Function	Periodic Green's Function
$g = \dfrac{e^{-jkR}}{R}$ $R = \lvert \vec{r} - \vec{r}' \rvert$	$g_{\text{periodic}} = \displaystyle\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{j\phi_{mn}} \dfrac{e^{-jkR_{mn}}}{R_{mn}}$ $R_{mn} = \sqrt{\left(x - x' - x_m\right)^2 + \left(y - y' - y_n\right)^2 + \left(z - z'\right)^2}$ $\phi_{mn} = k\left(x_m \sin\theta\cos\varphi + y_n \sin\theta\cos\varphi\right)$ $x_m = m \cdot d_x, \quad y_n = n \cdot d_y$

d_x and d_y are the ground plane dimensions that define the x and y dimensions of the unit cell. θ and Φ are the scan angles.

Comparing the two Green's functions, you observe an additional exponential term that is added to the infinite sum. The Φ_{mn} accounts for the scanning of the infinite array. The periodic Green's function also accounts for the effect of mutual coupling.

For more information see, "Infinite Arrays" on page 2-20.

References

[1] Harringhton, R. F. *Field Computation by Moment Methods*. New York: Macmillan, 1968.

[2] Rao, S. M., D. R. Wilton, and A. W. Glisson. "Electromagnetic scattering by surfaces of arbitrary shape." *IEEE. Trans. Antennas and Propagation*, Vol. AP-30, No. 3, May 1982, pp. 409–418.

[3] Wilton, D. R., S. M. Rao, A. W. Glisson, D. H. Schaubert, O. M. Al-Bundak. and C. M. Butler. "Potential Integrals for uniform and linear source distribution on polygonal and polyhedral domains." *IEEE. Trans. Antennas and Propagation*. Vol. AP-30, No. 3, May 1984, pp. 276–281.

[4] Balanis, C.A. *Antenna Theory. Analysis and Design*. 3rd Ed. New York: John Wiley & Sons, 2005.

More About

- "Infinite Arrays" on page 2-20
- "Method of Moments – Metal" on page 3-2